MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF ARTS

STA 401: BAYESIAN STATISTICS

DATE: 2/5/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Questions ONE and any other TWO questions

QUESTION ONE (COMPULSORY)(30 MARKS)

a) Explain the following terms as used in Decision Theory [3 Marks]

i) Posterior Probability

ii) Bayes theorem

iii) Conditional probability

b) Suppose there is a school with 60% boys and 40% girls as its students. The female students wear trousers or skirts in equal numbers; the boys all wear trousers. An observer sees a (random) student from a distance, and what the observer can see is that this student is wearing trousers. What is the probability this student is a girl?

[5 Marks]

c) In a certain place it rains on **one third** of the days. The local evening newspaper attempts to predict whether or not it will rain the following day. **Three quarters** of rainy days and **three fifths** of dry days are correctly predicted by the previous evening's paper. Given that this evening's paper predicts rain, what is the probability that it will actually rain tomorrow? (5 marks)

d) A machine is built to make mass-produced items. Each item made by the machine has a probability p of being defective. Given the value of p, the items are independent of each other. Because of the way in which the machines are made, p could take one of several values. In fact p = $x/100$ where x has a discrete uniform distribution on the interval [0, 5].The machine is tested by counting the number of items made before a defective is produced. Find the conditional probability distribution of x given that the first defective item is the thirteenth to be made. (5 marks)

e) There are five machines in a factory. Of these machines, three are working properly and two are defective. Machines which are working properly produce articles each of which has independently a probability of 0.1 of being imperfect. For the defective machines this probability is 0.2. A machine is chosen at random and five articles produced by the machine are examined. What is the probability that the machine chosen is defective given that, of the five articles examined, two are imperfect and three are perfect? (6 marks)

f) Demand x crates per day at Mama Njeri's Kiosk is given by the continuous probability density function

$$f(x) = \begin{cases} 1/10, & 0 \leq x \leq 10 \\ 0, & otherwise \end{cases}$$

The expected shortage quantity should not exceed one crate and the expected excess quantity should not exceed 2 crates. Determine the feasible milk stock level for this kiosk (6 marks)

QUESTION TWO (20 MARKS)

In a small survey, a random sample of 50 people from a large population is selected. Each person is asked a question to which the answer is either "Yes" or "No." Let the proportion in the population who would answer "Yes" be θ. Our prior distribution for θ is a beta (1.5, 1.5) distribution. In the survey, 37 people answer "Yes."

a) Find the prior mean and prior standard deviation of θ (4 marks)

b) Find the likelihood. (6 marks)

c) Find the posterior distribution of θ (3 marks)

d) Find the posterior mean and posterior standard deviation of θ (7 marks)

QUESTION THREE (20 MARKS)

The Aberdare lodge must decide on the level of supplies it must stock to meet the needs of its customers this December. The exact number of customers is not known but is expected to be in one of the four categories. 150, 200, 250 or 300. Four levels of supplies are thus suggested with level *i* being ideal if the number of customers falls in category *i*. The associated costs in terms of thousands of shillings are as follows.

Customer Category					
		ϴ1	**ϴ2**	**ϴ3**	**ϴ4**
Supplies levels	**A1**	10	15	23	30
	A2	13	12	13	28
	A3	26	23	17	26
	A4	35	27	24	20

a) Laplace Principle (5 marks)
b) Minimax (Maxmin) criterion (5 marks)
c) Savage Minimax Regret criterion (5 marks)
d) Hurwicz criterion with $\alpha=1/3$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Most (80%) of the taxis in Simple town are green, with the rest (20%) being yellow. In a traffic accident involving a hit-and-run taxi, a witness claims the taxi was yellow. Careful testing shows that the witness can successfully identify the color of a taxi only 75% of the time due to bad eyesight. On the balance of probabilities, should we hold Yellow Taxi Company liable? (4 marks)

b) In Nairobi Software Ltd, a computer in a group of 50 computers is serviced when it breaks down. At the end of T years, preventive maintenance is performed by servicing all 50 computers. The cost of repairing a broken computer is Kshs. 10,000 and the preventive maintenance cost per computer is Kshs. 1000. For T=1, 2,…,5 the probability that a computer will break down in period T is PT=0.05, 0.07, 0.10, 0.13, 0.18 respectively.

i) Use expected value criterion to determine the optimal decision for Nairobi software Ltd. (6 marks)

ii) For K=1, us the expected value-variance criterion to determine the optimal decision (10 marks)

QUESTION FIVE (20 MARKS)

Samples are taken from twenty wagon loads of an industrial mineral and analyzed. The amounts in ppm (parts per million) of an impurity are found to be as follows.

44.3 50.2 51.7 49.4 50.6 55.0 53.5 48.6 48.8 53.3

59.4 51.4 52.0 51.9 51.6 48.3 49.3 54.1 52.4 53.1

We regard these as independent samples from a normal distribution with mean μ and variance $\sigma^2 = \tau^{-1}$.

Find a 95% posterior hpd interval for μ under each of the following two conditions.

a) The value of τ is known to be 0.1 and our prior distribution for μ is normal with mean 60.0 and standard deviation 20.0 (10 narks)

b) The value of τ is unknown. Our prior distribution for τ is a gamma distribution with mean and standard deviation 0.05. Our conditional prior distribution for μ given τ is normal with mean 60.0 and precision 0:025τ (that is, standard deviation $\sqrt{40\tau^{1/2}}$): (10 marks)