MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ARTS

SMA 201: CALCULUS III

DATE: 6/5/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)(30 MARKS)

a) State the following theorems

i) Mean value theorem (2 marks)

ii) Rolle's theorem (2 marks)

iii) Taylor's theorem with Schlomilch form of remainder (2 marks)

iv) Maclaurin's theorem with Cauchy's form of remainder (2 marks)

b) State the L'Hospital's rule (2 marks)

c) Find the limit $\lim_{x \to 3} \frac{x^2 - 9}{x + 3}$ using L'Hospital's rule and verify the results by factoring

(3 marks)

d) Find the Maclaurin's series for $f(x) = \frac{1}{1+x}$ (3 marks)

e) Examine the validity of the hypothesis and conclusion of the specified theorem in each case below.

i) $f(x) = 1-(x-1)^{\frac{2}{3}}$ on $[0,2]$, (Rolle's theorem) (3 marks)

ii) $f(x) = x(x-1)(x-2)$ on $\left[0, \frac{1}{2}\right]$, (Mean Value Theorem) (3 marks)

f) Expand, if possible, $f(x) = \exp(x)$ in ascending powers of x upto the fourth term. (4 marks)

g) Obtain the first partial derivatives for

i) $f(x,y) = x^3 y + e^{xy^2}$ (2 marks)

ii) $f(x,y) = 2x^2 - xy + 2y^2$ at the point $(1,2)$ (2 marks)

QUESTION TWO (20 MARKS)

a) If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$, show that both partial derivatives exist at $(0,0)$ but the function is not continuous thereat. (5 marks)

b) State the Euler's theorem on homogeneous functions (2 marks)

c) Verify Euler's theorem for the function $f(x,y) = 4x^3 - 3x^2 y + 2xy^2 - 15y^3$. (3 marks)

d) Evaluate $\iint_R (x+y)\,dxdy$ where R is a rectangle $0 \le x \le 4$, $0 \le y \le 2$ (5 marks)

e) Find the extrema for the function $f(x,y) = 2x^2 - 2y^2 - x^4 + y^4$ (5 marks)

QUESTION THREE (20 MARKS)

a) State the Stoke's theorem (2 marks)

b) Verify stoke's theorem on $\mathbf{F} = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ taking $\hat{n}$ to be he unit vector perpendicular to the area bounded by the curve c in the xy-plane. (8 marks)

c) Find several normal vectors at the point $(2,1)$ on the curve $x^2 + y^2 = 5$ (5 marks)

d) Find $\frac{dy}{dx}$ on the curve $x^3y - xy^3 = 6$ at the point $(2,1)$ (5 marks)

QUESTION FOUR (20 MARKS)

a) State the Green's theorem (2 marks)

b) Evaluate the line integral given below using Green's theorem $\oint_c y^2dx + x^2dy$ where c is the square with vertices $(0,0), (1,0), (1,1), (0,1)$ oriented counterclockwise. (8 marks)

c) Evaluate the integral $\iiint_v xyzdxdydz$ over a domain bounded by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$ (5 marks)

d) Estimate the change in the value of $\frac{1}{\left(x^2 + y^2\right)^{\frac{1}{2}}}$ when (x, y) change from $(3, 4)$ to $(3.1, 3.8)$

(5 marks)

QUESTION FIVE (20 MARKS)

a) Obtain the first four Taylor polynomials for $lnx\ about\ x = 2$ (5 marks)

b) Obtain formulae for the Taylor polynomials for the following fun ctions centered at $x = a$ as far as $(x-a)^3$, stating the coefficient of $(x-a)^2$ in each case

i) $f(x)$

ii) $F(x) = f(x)g(x)$ (5 marks)

c) Obtain the Taylor's expansion for $f(x) = \cos x$ about $x = 0$

Using the above expansion, obtain the first three non-zero terms of the Taylor expansion for $\sec x$

Deduce the expansion for $f(x) = \cos\frac{1}{x-5}$ (10 marks)