MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 230: VECTOR ANALYSIS.

DATE: 29/4/2019 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer Question One And Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find the directional derivative of $\psi = (x+2y+z)^2 - (x-y-z)^2$ at the point (2,1,-1) in the direction of $\boldsymbol{A} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$. (4 marks)

b) A vector field $\mathbf{B}$ is given by $\mathbf{B} = \left(x^2 + xy^2\right)\mathbf{i} + \left(y^2 + x^2 y\right)\mathbf{j}$. Show that the field is irrotational (4 marks)

c) Given that $\vec{R} = e^{-t}\boldsymbol{i} + \ln(t^2+1)\,\boldsymbol{j} + \tan t\,\boldsymbol{k}$, determine;

i) $\left|\frac{d\vec{R}}{dt}\right|$ at $t = o$ (2 marks)

ii) $\left|\frac{d^2\vec{R}}{dt^2}\right|$ at $t = 0$ (2 marks)

d) Find the directional derivative $\phi = x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction

$2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ (4 marks)

e) State the following

i) Frenet serret formulae (3 marks)

ii) Green's theorem (2 marks)

iii) Stokes theorem (2 marks)

f) If $\mathbf{F} = x^2\mathbf{i} - xy\mathbf{j}$, evaluate $\int \mathbf{F} \bullet d\mathbf{R}$ from $(0,0)$ to $(1,1)$ along a straight line joining the two points. (4 marks)

g) If $\frac{d\mathbf{A}}{\mathbf{d}t} = \mathbf{C} \times \mathbf{A}$ and $\frac{d\mathbf{B}}{dt} = \mathbf{C} \times \mathbf{B}$, prove that $\frac{d}{dt}(\mathbf{A} \times \mathbf{B}) = \mathbf{C} \times (\mathbf{A} \times \mathbf{B})$ (3 marks)

QUESTION TWO (20 MARKS)

a) If the vector field $\mathbf{F} = 2y\mathbf{i} - z\mathbf{j} + x\mathbf{k}$,

i) Determine whether the point $P(1,0,2)$ is a source, sink or neither. (3 marks)

ii) Evaluate $\int_C \mathbf{F} \times d\mathbf{R}$ along a curve $x = \cos t$, $y = \sin t$ and $z = 2\cos t$ from $t = 0$ to $t = \frac{\pi}{2}$ (7 marks)

b) Show that the vector field $\mathbf{F} = (x^2 - yz)\mathbf{i} + (y^2 - xz)\mathbf{j} + (z^2 - xy)\mathbf{k}$ is irrotational. Find a scalar function ϕ such that $\mathbf{F} = \nabla\phi$. (10 marks)

QUESTION THREE (20 MARKS)

a) A particle moves along the curve $x = 2t^2, y = t^2 - 4\ and\ z = 3t$, determine the components of its velocity and acceleration at a time $t = 1$ in the direction of $\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ (7 marks)

b) Given that $\vec{A} = x^2yz\boldsymbol{i} - 2xz^3\boldsymbol{j} + xz^2\boldsymbol{k}$ and $\vec{B} = 2z\boldsymbol{i} + y\boldsymbol{j} - x^2\boldsymbol{k}$, determine $(\vec{B} \times \vec{A})$ at $(1,0,-2)$ (7marks)

c) Verify Green's theorem in the plane for $\oint_C [(x^2 + y^2)dx - 2xydy]$, where C is the rectangle bounded by $y = 0$, $x = 0$, $y = b$, $x = a$ (6 marks)

QUESTION FOUR (20 MARKS)

a) If $\mathbf{F} = (5xy - 6x^2)\mathbf{i} + (2y - 4x)\mathbf{j}$, evaluate $\int_c \mathbf{F} \bullet d\mathbf{R}$ along a curve C in the xy plane, $y = x^3$ from point $(1,1)$ to $(2,8)$. (5 marks)

b) If $|\mathbf{a}| = 13$, $|\mathbf{b} + \mathbf{a}| = 16$ and $|\mathbf{b} - \mathbf{a}| = 14$, find $|\mathbf{b}|$ (5 marks)

c) Verify the Stoke's theorem for the function $\mathbf{F} = \mathbf{3}xy\mathbf{i} - \mathbf{y}^2\mathbf{j}$ integrated over the curve $y = 2x^2$ in the $xy-$plane from $(0,0)$ to $1,2$ (10 marks)

QUESTION FIVE (20 MARKS)

Given the space curve defined by the parametric equations $x(t) = -\frac{-t^3}{3}$, $y(t) = t^2$, $z(t) = \frac{t^3}{3}$, find:

i) The unit tangent (4 marks)

ii) The principal normal (4 marks)

iii) The binormal (4 marks)

iv) The curvature (4 marks)

v) The torsion (4 marks)