



# MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)  
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND SEMESTER EXAMINATION FOR DEGREE IN BACHELOR EDUCATION

SMA 260: PROBABILITY & STATISTICS 1

DATE: SCHOOL BASED

TIME:

---

## INSTRUCTIONS:

*Answer Question ONE which is compulsory and any other TWO Questions Time 2 Hrs*

### QUESTION ONE (30 MARKS)

- (a) By citing examples differentiate between discrete and continuous random variable (4 marks)
- (b) The age of University staff members is normally distributed where 31% of the staff is below 45 years and 8% are over 64 years. Determine the mean and the standard deviation of the age distribution. (5 marks)
- (c) Given that  $X$  is a random variable with *pdf*

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & 1 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the quartile deviation of the random distribution (4 marks)

- (d) Given that  $F(x)$  is the CDF of a poisson distribution with parameter  $\lambda$  and that  $F(2) = 2F(1)$ . Determine the value of  $\lambda$  (6 marks)

(e) If  $X$  is a hyper geometric random variable, show that  $E(x) = \left(\frac{A}{N}\right)n$  (6 marks)

(f) Suppose the unit rate of receiving calls at an exchange board is 10 calls per hour.

Determine in three hour interval;

- i. The expected number of calls and its variance
- ii. The probability of receiving exactly 20 calls (5 marks)

### QUESTION TWO (20 MARKS)

(a) Suppose  $X$  is a random variable with a density function defined as

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \quad \lambda > 0 \\ 0 & , \quad otherwise \end{cases}$$

Determine the  $mgf_x$ ,  $E(x)$  and  $Var(x)$  (10 marks)

(b) The  $pdf$  of a random variable  $X$  is given by

$$f(x) = \begin{cases} \frac{1}{3} & , -1 < x < 2 \\ 0 & otherwise \end{cases}$$

Calculate  $E(x^2)$ ,  $E(x^3)$  and  $\mu_3$  (10 marks)

### QUESTION THREE (20 MARKS)

a) The lifespan of bulbs manufactured by a certain company is normally distributed with  $\mu = 2040$  hours and  $\sigma = 60$  hours. This is based on an experiment of 20,000 bulbs. Based on the experiment determine the

- i. Number of bulbs with a lifespan of 1,960 to 2,150 hours
- ii. Probability that a bulb picked at random would last for at least 2,200 hrs (6 mks)

b) An examination has 10 multiple-choice questions, with 4 choices for each question but only one is correct. To pass a student must answer at least 6 questions correct. If an ill prepared student decided to select the answers randomly determine the probability that the student

- i. Got no answer correct
- ii. Passed the examination (6 marks)

- c) If  $X$  is the number heads that show up after tossing a fair coin twice, determine the  $mgf_x$ ,  $E(x)$  and  $Var(x)$  (8 marks)

**QUESTION FOUR (20 MARKS)**

- a) Given that 20% of the articles taken from a batch of eight are found to be defective. Calculate the probability of getting three or more defective (4 marks)

- b) A variable  $X$  has a hypergeometric distribution with parameters  $A = 8$  and  $B = 20$ , for what value of  $n$ , will  $Var(X)$  be maximum (8 marks)

- c) The density for  $X$ , the lead concentration in gasoline in grams per liter is given by

$$f(x) = \begin{cases} 12.5x - 1.25 & , 0.1 \leq x \leq 0.5 \\ 0 & , \text{Otherwise} \end{cases}$$

Determine the

- i. Expected value of  $x$
- ii. Variability from liter to liter  $Var(x)$
- iii. Cumulative distribution function of  $x$  (8 marks)

**QUESTION FIVE (20 MARKS)**

- a) A container has 120 alarms of which 5 are defective. If 3 of these alarms are randomly selected and delivered to a customer, determine the probability that the customer will get one defective alarm. (4 marks)
- b) i) Highlight four properties of a binomial distribution function (4 marks)
- ii) If  $X$  is a binomial random variable then  $Var(x) = npq$  show (12 marks)