MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)

University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

Second Year Second Semester School Based (April-August 2016)

SMA 203: LINEAR ALGEBRA II

Date: …………………. **Time: ………………**

Instructions to Candidates

Answer question ONE and any other TWO questions

<u>QUESTION 1: 30 MARKS (Compulsory)</u>

a) Find the value of the following determinant

$$\begin{vmatrix} 2 & -2 & 3 \\ 1 & 4 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$ (3 marks)

b) Without expanding and by use of the properties of determinants show that the following determinants vanishes.

i. $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$ ii. $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ (4 marks)

c) i) Show that the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$ (4 marks)

ii. Using the expression in C(i) above evaluate A^{-1} and solve the equations

$x + 2y = 3$

$3x + y = 4$ (4 marks)

d) Show that the transformation

$T : R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x + 2y, 3y - 4z)$ is a linear transformation. (3 marks)

e) Find the Range, Nullspace and verify the Rank-Nullity theorem. (4 marks)

f) Find the characteristic equation and the eigenvalues for the matrix

$A = \begin{bmatrix} 5 & 4 \\ 4 & -1 \end{bmatrix}$ (4 marks)

g) For each eigenvalue, find the corresponding eigenvector and find a matrix p such that

$B = P^{-1}AP$ where is B is a diagonal matrix. (4 marks)

QUESTION 2: (20 MARKS)

a) Show that the mapping $T: R^3 \rightarrow R^2$ define by

$T(x, y, z) = (2x - y, y - 5z)$ is a linear transformation. (4 marks)

b) Find the matrix of the linear transformation with respect to the standard basis of R^3 i.e.

$\{(1,0,0)\ ,\ (0,1,0)\ , (0,0,1)\}$, in (a) above. (5 marks)

c) Write down a basis for the image of R^3 under T. (2 marks)

d) Find the Kernel of T. (4 marks)

e) Consider the polynomial $f(x) = 2 + 3x - 4x^2 + 2x^3$. Find the co-ordinates of $f(x)$ with respect to the basis $\{1\ ,\ (1 + x)\ ,\ (1 + x + x^2)\ ,\ (1 + x + x^2 + x^3)\}$ (5 marks)

QUESTION 3 (20 MARKS)

a) Describe the Cayley Hamilton theorem. (2 marks)

b) Find the characteristics equation for the matrix $T = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 5 & 0 & -1 \end{bmatrix}$ (6 marks)

c) Verify that the matrix T satisfies the characteristics equation and hence find T^{-1} (the inverse of T). (6 marks)

d) Find a matrix p that diagonalizes the matrix T and determine the product $P^{-1}\ TP$. (6 marks)

QUESTION 4 (20 MARKS)

a) Find a basis for the subspace W of R^4 generated by the set

$\{(1,2,3,1),(4,1,3,2),(6,5,9,4)\}$ (4 marks)

b) Express each of the given vectors in terms of the basis vectors obtained in (a) above. State the dimension of W. (6 marks)

c) Let A be the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 & 2 \\ 4 & 1 & 3 & 3 & 5 & 3 \\ 6 & 7 & 12 & 0 & 0 & 4 \\ 5 & 1 & 3 & 6 & 7 & 5 \end{bmatrix}$$

i. Use elementary row operations to reduce the matrix A to form $\begin{bmatrix} I & B \\ O & O \end{bmatrix}$ where I is the 3 by 3 identify matrix and O denote the 1 by 3 zero matrix.

ii. Write down a basis for the row space of A. (3 marks)

QUESTION 5 (20 MARKS)

A linear transformation $T: R^3 \rightarrow R^4$ is given by

$T(x,y,z) = ((x+y+3z), 2(y+z), 3(x+y+3z), 4(x+y+3z))$

Determine:

a) The matrix of T with respect to the standard bases of R^3 and of R^4. (5 marks)

b) The kernel of T. (4 marks)

c) The basis of the Kernel (3 marks)

d) The basis of the image (3 marks)

e) The rank of T. (2 marks)

f) The set of all vectors $\boldsymbol{v} \in R^3$ such that $T\boldsymbol{v} = (1,0,3,4)$. (3 marks)