MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTCS & PROGRAMMING.

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 330-NUMERICAL ANALYSIS

DATE: 4/8/2016 **TIME: 8:30 – 10:30 AM**

INSTRUCTION TO CANDIDATES

ANSWER **QUESTION ONE** AND **ANY TWO** OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Convert the decimal number 81.5 to its binary form. (2 marks)

b) Show that the operators μ and E commute. (3 marks)

c) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)

d) Prove that the forward difference of the quotient of two functions is given by

$$\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)}$$ (5 marks)

e) Set up a Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)

f) Find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4}

(8 marks)

QUESTION TWO (20 MARKS)

a) If E, μ and δ denote shift, average and central difference operators, in analysis of data with equal spacing h, prove the following

$$E^{1/2} = \mu + \frac{\delta}{2}$$

(2 marks)

b) Solve the equation $x^3 = Sinx$. Considering various $\phi(x)$, discuss the convergence of the solution. (5 marks)

c) For the following table of values, estimate $f(7.5)$, using Newton's backward difference interpolation formula

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	572

(5 marks)

d) Using the values given in the following table, find $Cos0.28$ by linear interpolation and by quadratic interpolation and compare the results with the value 0.96106 (exact to 5D)

x	0.0	0.2	0.4
$f(x)$	1.0000	0.98007	0.92106

(8 marks)

QUESTION THREE (20 MARKS)

a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

b) Find the positive solution of the transcendental equation

$2Sinx = x$ (5 marks)

c) Find the cubic polynomial which takes the following values; $f(1)=24,\ f(3)=120,\ f(5)=336,$ and $f(7)=720.$ hence or otherwise, obtain the value of $f(8)$. (5 marks)

d) Evaluate $\int_0^1 e^{-x^2}\,dx$ by means of Trapezoidal rule with $n=10$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Construct the forward difference table, where $f(x)=\frac{1}{x}, x=1(0.2)2,4D$ (4 marks)

b) Find $\int_0^{10}\frac{1}{1+x^2}dx$ using Simpson's one third rule. (7 marks)

c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FIVE (20 MARKS)

a) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation $f(x)=0$. (10 marks)

b) Use the method in a.) to find the root of the equation $\tan x = x$ near $x=4.5$ correct to four decimal places. (10 marks)