



MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND SEMESTER EXAMINATION FOR DEGREE IN

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 432-PARTIAL DIFFERENTIAL EQUATION I

DATE: 11/8/2016

TIME: 8:30 – 10:30 AM

INSTRUCTION TO CANDIDATES

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Solve $2yzdx + zxdy + xy(1+z)dz = 0$ (5 marks)

b) Verify the equation $(x+z)dx + zx^2dy + (yx^2 - x)dz = 0$ for integrability and solve it. (5 marks)

c) Eliminate the arbitrary function f from the equation

$$f(x^2 + y^2 + z^2, z^2 - 2xy) = 0 \quad (6 \text{ marks})$$

d) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$. (7 marks)

- e) Find the characteristics and the corresponding transport equations of the system

$$xyu_x - v_y = 0$$

$$xu_y - v_x = 0 \quad (7 \text{ marks})$$

QUESTION TWO (20 MARKS)

- a) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = c$. (5 marks)

- b) Find the integral curves of the equations

$$\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2} \quad (6 \text{ marks})$$

- c) Show that the Charpit's equation of the differential equation

$$(q^2 + 1)z^2 = 2pxz + x^2$$

Have an integral $qz = ax$ and find the corresponding complete integral of the equation.

(9 marks)

QUESTION THREE (20 MARKS)

- a) Show whether or not the set of equations

$$\left. \begin{aligned} x &= u + v \\ y &= u - v \\ z &= 4uv \end{aligned} \right\} \text{ represents a surface and if so, find the constraint equation.} \quad (10 \text{ marks})$$

- b) Determine the condition for the line

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n} \text{ to touch the quadric } \alpha x^2 + \beta y^2 + \gamma z^2 = 1. \quad (10 \text{ marks})$$

QUESTION FOUR (20 MARKS)

a) Find the integral surface of the curve given as

$$x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = z(x^2 - y^2) \quad (10 \text{ marks})$$

b) Find the equation of

i) The tangent line

ii) The normal plane to the curve

$$\left. \begin{array}{l} 3x^2 + y^2z + 2 = 0 \\ 2xz - x^2y - 3 = 0 \end{array} \right\} \text{at the point } p(1,-1,1). \quad (10 \text{ marks})$$

QUESTION FIVE (20 MARKS)

Verify the equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable and hence find its primitive (solution). (20 marks)