Machakos University College

(A Constituent College of Kenyatta University)

UNIVERSITY EXAMINATIONS 2013/2014

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL/ELECTRONICS ENGINEERING

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: Tuesday 1st April, 2014 TIME: 2 Hours

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO OF THE REMAINING QUESTIONS

QUESTION 1: COMPULSORY (30 MARKS)

a) Prove that

$$\frac{d}{dx}\left(J_o(x)\right) = -J_1(x)$$ (8 marks)

b) Suppose F is such that the Laplace transform $\mathcal{L}\{F(t)\}$exists for $s > \alpha$.Then for any constant a, prove that

$$\mathcal{L}\{e^{at}F(t)\} = f(s-a)$$

Where $s > \alpha + a$ and $f(s)$ denotes $\mathcal{L}\{F(t)\}$. Hence or otherwise find $\mathcal{L}\{e^{at}t\}$ (8 marks)

c) Determine inverse Laplace transform of the function

$$f(s) = \frac{1}{s(s^2-1)}$$

(6 marks)

d) Solve the partial differential equation

$$\frac{\delta^2 u}{\delta x^2} - \frac{\delta^2 u}{\delta x \delta y} - 6\frac{\delta^2 u}{\delta y^2} = x + y$$

(8 marks)

QUESTION 2 (20 MARKS)

a) Using the series definition for J_p, show that

i. $\frac{d}{dx}\left[x^p J_p(kx) = kx^p J_{p-1}(kx)\right]$

ii. $\frac{d}{dx}\left[x^{-p} J_p(kx) = -kx^{-p} J_{p+1}(kx)\right]$

where k is a constant. (8 marks)

b) Show that the alternative solution of the Bessel differential equation in terms of $J_n(x)$ is

$$y = AJ_n + BJ_n \int \frac{1}{xJ_n^2} dx$$

where A and B are arbitrary constants. (12 marks)

QUESTION 3 (20 MARKS)

a) Let F be a real valued function which is continuous for $t \geq 0$ and of exponential order $e^{\alpha t}$. If F', the derivative of F, is a piecewise continuous in every finite closed interval $a \leq t \leq b$, then prove that the Laplace transform of $F'(t)$ is given by $\mathcal{L}\{F'(t)\} = s\mathcal{L}\{F(t)\} - F(0)$ for $s > \alpha$ (12 marks)

b) Find the inverse Laplace transform of the function

$$f(s) = \frac{1}{s^2 + 6s + 13}$$

using Laplace transform table. (8 marks)

QUESTION 4 (20 MARKS)

a) Find the trigonometric Fourier series of the function f defined by

$$f(x) = |x|,$$

on the interval $-\pi \leq x \leq \pi$ (8 marks)

b) Consider the function f defined by

$f(x) = 2x$, $0 \leq x \leq \pi$

Find

i. the Fourier sine series of f on $0 \leq x \leq \pi$ and

ii. the Fourier cosine series of f on $0 \leq x \leq \pi$

(12 marks)

QUESTION 5 (20 MARKS)

Solve he partial differential equation

$$\alpha^2 \frac{\delta^2 y}{\delta x^2} = \frac{\delta^2 y}{\delta t^2}$$

Where $\alpha^2 = \frac{\tau}{\rho}$ and y satisfies the initial conditions

$$y(0,t) = 0, \qquad 0 \le t \le \infty$$

$$y(\pi,t) = 0, \qquad 0 \le t \le \infty$$

$$y(x,0) = \sin 2x\,, 0 \le x \le \pi$$

$$\frac{\delta y}{\delta t}(x,0) = 0, \qquad 0 \le x \le \pi$$

by the method of separation of variables. (20 marks)

****