Machakos University College

(A Constituent College of Kenyatta University)

UNIVERSITY EXAMINATIONS 2013/2014

SCHOOL OF COMPUTING AND APPLIED SCIENCES

SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTING AND APPLIED SCIENCES

SMA 230: VECTOR ANALYSIS

DATE: 9/4/2014 **TIME:** 8.30 a.m. – 10.30 a.m.

Instructions: Answer question ONE and any other TWO questions.

Question 1

(a) Find the projection of $\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}$in the direction of the vector $\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$.(3mks)

(b) Find the equation of the plane through the points $\boldsymbol{P_1}(2,-1,1), \boldsymbol{P_2}(3,2,-1)$, and $\boldsymbol{P_3}(-1,3,2)$ (4mks)

(c) If $\boldsymbol{A}=x^2yz\boldsymbol{i}-2xz^3\boldsymbol{j}+xz^2\boldsymbol{k}$

$$\boldsymbol{B}=2z\boldsymbol{i}+4y\boldsymbol{j}-x^2\boldsymbol{k}$$

Find

$\frac{\partial^2}{\partial x \partial y}(\boldsymbol{A}x\boldsymbol{B})at(1,0,-2)$ (5mks)

(d)

i. When is a vector said to be irrotational? (1mk)

ii. Find the constants a,b,c so that

$$\boldsymbol{V}=(x+2y+az)\boldsymbol{i}+(bx-3y-z)\boldsymbol{j}+(4x+cy+2z)\boldsymbol{k}$$

Is irrotational. (4mks)

(e) If $\boldsymbol{A}=(3x^2+6y)\boldsymbol{i}-14yz\boldsymbol{j}+20xz^2\boldsymbol{k}$, evaluate

$\int_c A.dr$ from$(0,0,0)$ to $(1,1,1)$ along the following paths c

i. $x=t, y=t^2, z=t^3$ (6mks)

ii. The straight line from $(0,0,0)$ to $(1,0,0)$ then to $(1,1,0)$ and then to $(1,1,1)$. $(4mks)$

iii. The straight line joining $(0,0,0)$ and $(1,1,1)$ (3mks)

Question 2

Find the volume of the parallelopiped

(a) With edges $\boldsymbol{i}+\boldsymbol{j}+\boldsymbol{k},\ \boldsymbol{j}+\boldsymbol{k}$ and $2\boldsymbol{k}$. (3mks)

(b) The acceleration of a particle at any time $t \geq 0$ is given by

$$\boldsymbol{a} = \frac{dv}{dt} = 12cos\ \ 2t\ \boldsymbol{i} - 8sin\ \ 2t\ \boldsymbol{j} + 16t\ \boldsymbol{k}$$

If the velocity,**V** and displacement,**r** are zero at $t = 0$. Find **V** and **r** at any time, t.(8mks)

(c) Verify the Green's Theorem in the plane for $\oint_c s(xy + y^{2)}dx + x^2dy$
where c is the closed curve if the region bounded by $y = x$ and $y = x^2$ (9mks)

Question 3

(a) Prove the diagonals of a parallelogram bisect each other. (5mks)

(b) If $R(u) = (u - u^2)\boldsymbol{i} + 2u^3\boldsymbol{j} - 3\boldsymbol{k}$ find ∇xF

i. $\int R(u)du$ (3mks)

ii. $\int_1^2 R(u)du$ (3mks)

(c)

i. Show that $\boldsymbol{F} = (2xy + z^{3)}\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\boldsymbol{k}$ is a conservative force field. (3mks)

ii. Find the scalar potential. (4mks)

iii. Find the work done in moving an object in the field from $(1,-2,1)$ to $(3,1,4)$. (2mks)

Question 4.

(a) If **a** and **b** are non-collinear vectors and

$$\boldsymbol{A} = (x + 4y)\boldsymbol{a} + (2x + y + 1)\boldsymbol{b}$$
$$\boldsymbol{B} = (y - 2x + 2)\boldsymbol{a} + (2x - 3y - 1)\boldsymbol{b}$$

Find x and $y$if $3\boldsymbol{A} = 2\boldsymbol{B}$ (5mks)

(b) State and prove the sine rule. (7mks)

(c) If $\boldsymbol{A} = xz^3\boldsymbol{i} - 2x^2yz\boldsymbol{j} + 2yz^4\boldsymbol{k}$.
Find $\nabla\ x\ \boldsymbol{A}$(or curl **A**) at the point $(1,-1,1)$. (4mks)

(d) If $\boldsymbol{A} = x^2z\boldsymbol{i} - 2y^3z^2\boldsymbol{j} + xy^2z^2\boldsymbol{k}$,
find$\nabla.\boldsymbol{A}$ (or div. **A**) at the point $(1,-1,1)$.(4mks)

QUESTION 5.

(a) Show that the acceleration of a particle moving along a space curve with velocity V is given by

$$a = \frac{dv}{dt} T + V^2 KN$$

where T is the unit tangent vector to the space curve. N the unit principal normal and K is the curvature

For a particle moving along a space curve X = 3cost t, Y = 3sin t, Z=4t. Find the normal and tangential components of the acceleration. (15 marks)

(b) State the Frenet-serret formulae. Find the curvature and torsion of the space curve in (a) above. (5 marks)