

## Machakos University College (A Constituent College of Kenyatta University) UNIVERSITY EXAMINATIONS 2013/2014 SCHOOL OF COMPUTING AND APPLIED SCIENCES SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE <u>SCO 213: NUMERICAL ANALYSIS</u>

DATE: 8/3/2014

**TIME:** 8.30 a.m. – 10.30 a.m.

### **INSTRUCTIONS:**

Answer question ONE which is Compulsory and any other TWO questions

#### **Question One (Compulsory)**

- (a) The true values of two quantities x and y are 4.00 and 1.00 respectively. If their computed values are 4.04 and 1.02 respectively determine the absolute error, relative error and percentage error in their product. (5 marks)
- (b) The function  $f(x) = e^x cosx$  is evaluated at x = 1. Find the error in f(x) if the x used in the computation has an error of 0.01. (5 marks)
- (c) A function takes on the following values:
  - $f_0 = 0, f_1 = 41, f_2 = 79, f_3 = a, f_4 = 146.$
  - (i) Express  $\Delta^4 f_0$  using functional values  $f_0, f_1, f_2, f_3, f_4$ , where  $\Delta$  is the forward difference operator. (3 marks)
  - (ii) If  $\Delta^4 f_0 = 0$  use the expression in (i) above and the values of the function given to find the value of a. (i.e.  $f_3$ ). (2 marks)

(d) Construct a backward difference table for  $f(x) = x^2 - 3x^2 + 5x - 7$  for x = -1, 0, 1, 2, 3, 4, 5. (5 marks)

(e) By letting h be the interval of differencing show that  $\left(\frac{\Delta^2}{E}\right)x^3 = 6xh$  (4 marks)

(f) Use the trapezium rule to estimate the area from x = 0.2 to x = 1 for the curve  $f(x) = x^2 + x$  using n = 4 (6 marks)

### **Question Two**

(a) A function takes the following values shown in the table below:

X	0	1	2	3	4	5	6
f(x)	5	6	13	32	69	130	221

- (i) Construct a forward difference table for the function. (7 marks)
  (ii) Using the constructed table find f(7) and f(8). (4 marks)
- (b) Given a function f(x) such that f(0) = -3, f(1) = -5, f(3) = 18. Find the 2 degree lagrange interpolating polynomial. (9 marks).

#### **Question Three**

- (a) Evaluate  $\int_0^{12} \frac{1}{1+x^2} dx$  by using Simpson's <sup>1</sup>/<sub>3</sub> rule taking n = 6. (8 marks)
- (b) Find the exact value of  $\int_0^{12} \frac{1}{1+x^2} dx$  and compute the percentage error introduced in (a) above. (5 marks)
- (c) Use Newton-Raphson method to find the real root near o of the equation  $x^2 + x 5 = 0$ to 2d.p. (7 marks)

#### **Question Four**

- (a) Given that  $\Delta$  is the forward difference operator and  $\nabla$  is the backward difference operator, show that  $(1+\Delta)(1-\nabla) = 1$  (4 marks)
- (b) Relate the shift operator E to the differential operator D starting with a taylor series using h as the step size. (12 marks)
- (c) Using (b) above deduce the relationship between the operator  $\Delta$  and D. (4 marks)

# **Question Five**

- (a) Use the bisection method to determine the root of  $f(x) = e^{-x} x$  giving your answer to 4 decimal places. (10 marks)
- (b) Apply lagranges interpolation formula to find apolynomial which passes through the points (0, -20) (1, -12) (3, -20) and (4, -24). (10 marks)