MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE IN

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION (SCIENCE)

SMA 361: THEORY OF ESTIMATIOIN

DATE: 12/8/2016 **TIME:8.30-10.30AM**

Answer Question One and Other Two Questions

QUESTION ONE (COMPULSORY)

(a) (i) Explain the meaning of the following terms as applied in theory of estimation
  (i) Population
  (ii) An estimator
  (iii) Consistency (3 marks)

(ii) Let $x_1, x_2, .. x_n$ be a random sample of size n from a distribution with mean from a population with mean μ and variance δ^2 , show that $\bar{x}$ is a consistent estimator of μ.

(6 marks)

(b) (i) A random sample of five values are taken from a normal population with mean μ and variance δ^2both unknown. The values are: 1.07, 1.02, 1.04, 1.06, 1.05. Estimate the population mean and population variance. (3 marks)

(ii) $Let\ x_1, x_2, .. x_n$ be a random sample from a population with mean μ and variance δ^2. Consider the following estimators for μ.

$$t_1 = {}^1\!/_2\,(x_1 + x_2)$$

$$t_2 = \frac{\frac{1}{2}x_1 + x_2 + \cdots + x_{n-1}}{2(n-1)}$$

$$t_3 = \bar{x}$$

- Show that each of 3 estimators is unbiased. (5 marks)
- Determine the efficiency of t_3 relative to t_2 (5 marks)

(c) Let $x_1, x_2, .. x_n$ denote a random sample from a pdf

$$f(x) = \begin{cases} (\theta + 1)x^{\theta}, & 0 < \mathrm{x} < 1,\ \ \theta > 0 \\ 0 & elsewhere \end{cases}$$

Obtain the moment estimator of θ (8 marks)

QUESTION TWO

(a) Let $x_1, x_2, .. x_n$ be a random sample from $x \sim N(\mu_0, \delta^2)$ where μ, δ^2 are both unknown. Determine a joint sufficient statistic of $\mu, and\ \delta^2$ (8 marks)

(b) Given a random sample of size n from a population whose pdf is

$$f(x) = \frac{1}{2\sqrt{2\pi\theta}} e^{\frac{-1}{4\theta}(x-5)^2}$$

Obtain the maximum likelihood estimator (MLE) of θ and show that it's unbiased. (12 marks)

QUESTION THREE

(a) Let $x_1, x_2, .. x_n$ be a random sample of size n from a population whose pdf if $f(x, \theta)$ where θ is unknown. Let $T = t(x_1, x_2, \ldots x_n)$ be the necessary and sufficient condition for T to be the Minimum variance unbiased estimator (MVUE) of θ expressed in the form

$$\frac{\partial LogL}{\partial \theta} = \frac{T - \theta}{\lambda}$$

Show that T is MVUE of θ and $Var(T) = \lambda$ (10 marks)

(b) A random sample of size n is drawn from a normal population with $(0,\delta^2)$. Determine the minimum variance unbiased estimator (MVUE) of δ^2 and its variance. (10 marks)

QUESTION FOUR

Three objects P, Q and R with weights Z_1, Z_2, and Z_3 respectively are weighed on a spring balance P and Q together weigh 75g, P and R weigh 45g and Q and R weigh 100g. Assuming the weights are independent with constant variance δ^2, find the least square estimates of Z_1, Z_2, and Z_3 (20 marks)

QUESTION FIVE

(a) A population has a known mean μ and a standard deviation δ of 2.45. If a sample of 900 has a mean of 3.15cm and the population is normal obtain a 98% confidence intervals for the true mean. (8 marks)

(b) If $x_1, x_2, .. x_n$ is a random sample of size n from a population whose pdf is given by

$$f(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

Determine the uniformly minimum variance unbiased estimator (UMVUE) of $e^{-\lambda}$

(12 marks)