MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATIONS FOR THE DEGREE IN

BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION ARTS

SMA 430: NUMERICAL ANALUSIS II

Date: 10/8/2016 **Time: 8:30 – 10:30 AM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Solve the system of equations below using the inverse of the coefficients matrix method (5 marks)

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$
$$3x_1 + 3x_2 + 2x_3 = 3$$

b) Determine the Eigen values and the corresponding Eigen vectors of the following system. (5 marks)

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

c) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$,evaluate $f'(x)$ (5 marks)

d) Compute the approximate value of $I = \int_{-1}^{1} \frac{dx}{1+x}$ using the Lobatto integration methods

(5 marks)

e) Consider the initial value problem (I .v. p) given by

$y' = 2x + 3y \quad ; \; y(0) = 1$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$

(5 marks)

f) Using Jacobi's iteration method, solve the system of equations below, perform two iterations (5 marks)

$$10x + 2y + z = 9$$
$$x + 10y - z = -22$$
$$-2x + 3y + 10z = 22$$

QUESTION TWO (20 MARKS)

Consider the differential equation $\frac{dy}{dx} = 2e^{x} y$, $y(0) = 2$. Calculate y(0.4) using Adams predictor corrector formula by calculating $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ using the Euler's modified formula.

(20 marks)

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_{-1}^{1} e^{-x^2} \cos x \, dx$ obtain the value using.

a) Gauss-Legendre integration method for $n = 2, 3$

(10 marks)

b) Radau integration method for $n = 2, 3$

(10 marks)

QUESTION FOUR (20 MARKS)

a) Calculate the largest Eigen –value for the matrix

$$\begin{bmatrix} 10 & 4 & -1 \\ 4 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Also evaluate the Eigen vector corresponding to the Largest Eigen -value

(10 marks)

b) Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Where a is a real constant

i) Determine the values of a, for which the Jacobi and Gauss-Seidel methods converge.

(7 marks)

ii) For $a = 0.5$ calculate the value of ω which minimizes the spectral radius of the SOR iteration method.

(3 marks)

QUESTION FIVE (20 MARKS)

a) Solve the system of equations below using Doolittle's method.

(8 marks)

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

b) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting (7 marks)

c) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Romberg integration with step size $h = \frac{1}{16}$ (5 marks)