MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF EDUCATION (SCIENCE, ARTS AND SPECIAL EDUCATION)
BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 260 - PROBABILITY AND STATISTICS I

DATE: 12/8/2016 **TIME: 2.00-4.00 PM**

***Instructions to the Candidate*:**

1. *Answer **Question 1** and any other **two** questions.*
2. *Out of the **three** questions answered, each question must start on a new page.*
3. *You must have the following items for this paper:*
   - *Statistical tables;*
   - *Scientific calculator.*

1. (a) Outline **three** characteristics of the binomial probability distribution, illustrating with an example from a real life situation for each characteristic. (3 *marks*)

   (b) (i) Given a continuous random variable x, state **two** conditions that must be satisfied for the function $f(x)$ to be considered a probability density function (p.d.f.). (2 *marks*)

   (ii) If x is a continuous random variable with a probability density function given by:

$$f(x) = \begin{cases} ax^2 ; & \text{for} \quad 1 \leq x \leq 2 \\ 0 \; ; & \text{otherwise} \end{cases}$$

Determine the value of the constant a. (3 *marks*)

(c) If x is a discrete random variable with a probability mass function $f(x)$ given by:

$$f(x) = \begin{cases} \frac{x}{6} \; ; & \text{for} \quad x = 1, 2, 3 \\ 0 \; ; & \text{otherwise} \end{cases}$$

Determine the moment generating function of x, and hence determine the mean and variance of x. (6 *marks*)

(d) The height of 3000 students in a certain university has been found to be normally distributed with mean 68 inches and standard deviation 12 inches. Determine the following about the distribution:

(i) The probability that a student selected at random has a height of between 60 and 70 inches; (4 *marks*)

(ii) The number of students whose height is between 60 and 70 inches. (1 *mark*)

(e) Given a continuous random variable x, and taking its variance, prove that:

$$E\left[(X-\mu)^2\right] = E(X^2) - \mu^2$$ (3 *marks*)

(f) A discrete random variable x has a Poisson probability distribution with probability mass function (p.m.f.) given by:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{for} \quad x = 1, 2, 3, \dots, \infty, \quad \text{and} \quad \lambda > 0.$$

Show that for the random variable x: $\mu = var(x) = \lambda$ (8 *marks*)

2. A continuous random variable x is given by the probability function

$$f(x) = \begin{cases} \frac{1}{36}(8x - x^2 - 7) \; ; & \text{for} \quad 1 \leq x \leq 7 \\ 0 \; ; & \text{elsewhere} \end{cases}$$

(a) Show that the function $f(x)$ is a probability density function (p.d.f.). (3 *marks*)

(b) Determine by calculation the following measures about the random variable x in the probability distribution:

(i) the mode of x ;

(ii) the mean of x ;

(iii) the variance of x . (10 *marks*)

(c) Determine the cumulative distribution function of x. Hence, determine the following:

(i) the median of x.

(ii) the probability: $P(x \leq 5)$. (7 *marks*)

3. (a) A random variable x has a binomial probability distribution. Derive the expectation and the variance of the random variable x (without using the moment generating function technique). (10 *marks*)

   (b) Research findings on loaves of bread produced in Machakos shows that 40% of the loaves actually expire before the indicated expiry date. A random sample of 12 loaves was taken. Determine the probability that among these loaves, the following will expire before the indicated expiry date:

   (i) exactly 6 loaves;

   (ii) at least 3 loaves;

   (iii) between 4 and 6 loaves inclusive. (10 *marks*)

4. (a) A continuous random variable x has an exponential probability distribution with probability density function (p.d.f.) given by:

$$f(x) = \begin{cases} \lambda\, e^{-\lambda x}\ ; & \text{for} \quad 0 \leq x \leq \infty\,, \qquad \lambda > 0. \\ 0 \quad \text{elsewhere} & \end{cases}$$

   Prove that the variance of x is given by: $var(x) = \dfrac{1}{\lambda^2}$ (10 *marks*)

   (b) It has been observed that 4 out of every 100 nails coming out of a manufacturing process are defective. A random sample of 240 nails is selected from the manufacturing process.

   (i) Derive the probability mass function (p.m.f.) for this distribution; (2 *marks*)

   (ii) Determine the probability that among the nails selected, the following will be defective:

   I. at least 3 nails;

   II. between 6 and 8 nails inclusive. (8 *marks*)

5. The marks scored in Mathematics by students who sat for the KCSE examination in the year 2015 is normally distributed with a mean of 52 marks and a standard deviation of 12 marks.

   (a) (i) Suppose the pass-mark is set at 42 marks, determine the proportion of the students who will pass. (3 *marks*)

   (ii) Determine the proportion of the students who will score a Grade C if the grade is assigned for marks between 56 and 64. (7 *marks*)

   (b) (i) If the top 68% of the students are supposed to pass this examination, determine the mark which should be set as the pass-mark to achieve this. (3 *marks*)

   (ii) Grades for results are awarded as follows:

   - *Fail* to the bottom 20%,
   - *Pass* to the next 40%,
   - *Credit* to the next 25%,
   - *Distinction* to the top 15%.

   Determine the lower and upper limits of the range of the marks for the grades Pass and Credit. (7 *marks*)

Statistical Formulae

μ = E(X) = $\sum_{i=1}^{n} x_i \, f(x_i)$

= $\int_0^{\infty} x f(x) \, dx$

Var(x) = E(X^2) − [E(X)]^2 = $\sum_{i=1}^{n} x_i^2 \, f(x_i) - \mu^2$

= $\int_a^b x^2 \, f(x) \, dx - \mu^2$

Binomial probability mass function f(x) = $\binom{n}{x} p^x (1-p)^{n-x}$ OR $\binom{n}{x} p^x q^{n-x}$

Mean = E[X] = $n\,p$ and var (x) = $n\,p$ (1 - p) = $n\,p\,q$

Poisson probability mass function $f(x)$ = $\dfrac{\lambda^x e^{-\lambda}}{x!}$ Mean = $n\,p$ = λ and var (x) = λ

Exponential probability density function $f(x)$ = $\lambda \, e^{-\lambda x}$ Mean = $\frac{1}{\lambda}$ and var (x) = $\frac{1}{\lambda^2}$

Normal probability density function $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

for $-\infty \le x \le \infty$, $-\infty \le \mu \le \infty$, $0 \le \sigma \le \infty$

The standardised value z for a normal random variable x: $z = \dfrac{x-\mu}{\sigma}$

Standard normal probability density function $f(z) = \dfrac{1}{\sqrt{2\pi}} \, e^{-\frac{1}{2}z^2}$ for $-\infty \le z \le \infty$,

where mean μ = 0 and standard deviation σ = 1