MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)
BACHELOR OF EDUCATION

SMA 202: LINEAR ALGEBRA 1

DATE: 1/8/2016 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer QUESTION ONE and Any other TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{bmatrix}$,

i) Define what is meant by a rank of a matrix (1 mark)

ii) Obtain the rank of the above matrix. (4 marks)

b) Find the matrix of the minors and cofactors of the for the matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$

(3marks)

c) Let $V = P_3$ and W be a set of all polynomials of degree 3 or less but with a constant zero term. Determine if W is a subspace of P_3. (5 marks)

d) Determine the point of intersection of following line and plane $\frac{x-2}{-1}=\frac{y+2}{3}=\frac{z-1}{5}$ and

$x-5y+2z=7$ respectively (5 marks)

e) Determine if $S=\{(1,2),(-1,1)\}$ is a basis for $\Re^2$ (5 marks)

f) Determine the unit vector perpendicular to the plane of $\mathbf{A}=2\mathbf{i}-6\mathbf{j}-3\mathbf{k}$ and $\mathbf{B}=4\mathbf{i}+3\mathbf{j}-\mathbf{k}$ (4 marks)

g) Find the length of the vector $\mathbf{U}=2\mathbf{i}-4\mathbf{j}+8\mathbf{k}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the equation below giving your answer in both parametric and vector form using Gauss elimination method. $\begin{matrix} 2a+b+c=4 \\ -a+2b-c=3 \end{matrix}$ (6 marks)

b) Given $V=\Re^4$ and $S=\{(1,-2,0,3),(2,3,0,-1),(2,-1,2,1)\}$, determine if $(3,9,-4,-2)\in L(s)$ i.e. the set spanned by S hence a subspace. (8 marks)

c) Given that T is a transformation defined as $T:\Re^3\rightarrow\Re^3$ such that $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 2x_1+x_2-x_3 \\ -x_3+x_2 \\ x_1-x_2-x_3 \end{pmatrix}$, find the matrix of T with respect to the standard basis (6 marks)

QUESTION THREE (20MARKS)

a) Reduce the following matrix in to row echelon form $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{bmatrix}$ (4 marks)

b) With the help of an example explain the nature of solutions that may arise whenever a system of equations is solved using row echelon method. (6 marks)

c) Given that $T:\Re^3\rightarrow\Re^3$ is defined as $T(x)=\begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$,

Find :

i) Basis for Rank of (T) (3 marks)

ii) Basis for the kernel of (T) (3 marks)

iii) Rank of (T) (1 mark)

iv) Nullity of (T) (1 mark)

d) Obtain the cross product of the vectors $\mathbf{A} = (1,2,-1)$ and $\mathbf{B} = (0,2,3)$. i.e. $\mathbf{A} \times \mathbf{B}$ (2 marks)

QUESTION FOUR (20MARKS)

a) Solve the following system of equations using the Crammer's rule
$$\begin{aligned} 2x_1 + x_2 - 2x_3 &= 10 \\ 3x_1 + 2x_2 + 2x_3 &= 1 \\ 5x_1 + 4x_2 + 3x_3 &= 4 \end{aligned}$$
(8 marks)

b) Find the inverse of the following matrix if possible.
$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$
(7 marks)

c) Determine the equation of the plane perpendicular to the line $\mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ which passes through the point $A(1,2,1)$ (5 marks)

QUESTION FIVE (20MARKS)

a) Find the area of the triangle having vertices at $P(1,3,2), Q(2,-1,1)$ and $R(-1,2,3)$ (6 marks)

b) Determine the solution to the plane $-x_1 + 2x_2 - 3x_3 = 4$ giving your answer in both parametric and vector form. (6marks)

c) Show that $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} \neq \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ given that $\mathbf{A} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{B} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{C} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ (5 marks)

d) Define the following in relation to vector spaces
   - i) Dimension (1 mark)
   - ii) Basis (1 mark)
   - iii) Kernel (1 mark)