



# MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)  
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DIPLOMA IN AUTOMOTIVE  
ENGINEERING  
DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION)

PDE MED 301: PARTIAL DIFFERENTIAL EQUATIONS

Date: 11/8/2016

Time: 2:00 – 4:00 PM

INSTRUCTIONS:

Answer Question one and any other two questions

## QUESTION ONE (COMPULSORY)

a) Show that each of the following is a parametric equation of

$$x^2 + y^2 + z^2 = a^2$$

i)  $x = a \frac{1-v^2}{1+v^2} \cos u, y = a \frac{1-v^2}{1+v^2} \sin u, z = \frac{2av}{1+v^2}$  (5 marks)

ii)  $x = a \sin u \cos v, y = a \sin u \sin v, z = a \cos u$  (5 marks)

b) Find the orthogonal trajectories in the cone  $x^2 + y^2 = z^2 \tan^2 \alpha$  of its intersection

with the family of the plane parallel to  $z=c$  (5 marks)

c) Show that the direction cosine of the tangent at the point  $(x,y,z)$  to the conic

$ax^2 + by^2 + cz^2 = 1, x + y + z = 1$  are proportional to  $(by - cz, cz - ax, ax - by)$  (6 marks)

d) Find the integral curves of the equation

$$\frac{dx}{y^2} = \frac{dy}{xy} = \frac{dz}{x(z-2y)} \quad (6 \text{ marks})$$

e) Given  $z=x^2+3xy+5y^2$ ,  $x=\sin t$ ,  $y=\cos t$

Find  $\frac{dz}{dt}$

(3 marks)

**QUESTION TWO: (20 MARKS)**

a) Find the orthogonal trajectories in the surface  $x^2+y^2+2fyz+d=0$  of its curves of intersection with planes parallel to the x-y plane (12 marks)

b) Find the integral curves of the equations  $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$  (8 marks)

**QUESTION THREE: (20 MARKS)**

a) Construct a partial differential equation by eliminating a,b and c from  $z=a(x+y) + b(x-y) + abt + c$  (10 marks)

b) A particle moves along a curve where parametric equations are  $x=e^t$ ,  $y=2\cos 3t$ ,  $z=2\sin 3t$ , t is the time

i) Determine the velocity and acceleration at anytime. (5 marks)

ii) Find the magnitude of the velocity and acceleration at time  $t=0$  (5 marks)

**QUESTION FOUR: (20 MARKS)**

a) Construct a partial differential equation by eliminating a and b from  $ax^2+by^2+z^2=1$  (8 marks)

b) If  $z=2x^2-3xy+4y^2$ , find  $z_x, z_y, z_{xx}, z_{yy}, z_{xy}, z_{yz}$  (12 marks)

**QUESTION FIVE: (20 MARKS)**

a) Find the total differential of  $z=x^2y+x^2y^2+xy^3$  (6 marks)

b) Approximate the change in the hypotenuses of a right angle triangle of length 60mm and 80mm when the shorter length is lengthened by 2.5mm and the longer length is shortened by 1.25mm (14 marks)