MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF STATISTICS AND PROGRAMMING

SMA 401: TOPOLOGY II

Date: 4/8/2016 **Time:**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Let A be a subset of a topological space X. Define the following terms:
   i. An open cover for a set A. (1 mark)
   ii. Finite subcover of a set A. (1 mark)
   iii. A compact set A in X. (2 marks)

b) State the Heine-Borel theorem. (2 marks)

c) Show in details that the open interval $A=(0,1)$ on $\Re$ with the usual topology is not compact. (4 marks)

d) When is a class $\{A_i\}$ of subsets of a topological space X said to have the finite intersection property? Give an example of a class that has the finite intersection property. (5 marks)

e) Let f be a one to one continuous function from a compact space X into a Hausdorff space Y. Prove that X and $f(x)$ are homeomorphic. (5 marks)

f) Let A be a subset of a topological space X. Define the following terminologies:
   i. Sequentially compact set A in X. (1 mark)
   ii. Countably compact set A in X. (1 mark)
   iii. Locally compact set X. (1 marks)

g) Show that a locally compact space need not be compact but that every compact space is locally compact. (3 marks)

h) Distinguish between a connected space and a space that can be separated. Hence give an example of a connected topological space. (4 marks)

QUESTION TWO (20 MARKS)

a) Prove that a continuous image of a compact set is also compact. (8 marks)

b) Let A be a subset of a topological space (X,τ). Prove that A is compact with respect to τ if and only if A is compact with respect to the relative topology τ_A on A. (12 marks)

QUESTION THREE (20 MARKS)

a) Show that if A is a connected subspace of a topological space X and $A \subset B \subset \overline{A}$, then B is also connected. (8 marks)

b) Let X be a topological space. Prove that the following statements are equivalent.
- X is compact.
- For every collection $\{F_i\}$ of closed subsets of X, $\bigcap_{i=1}^{\infty} F_i$ implies that $\{F_i\}$ contains a finite subcollection $\{F_{i_1}, F_{i_2}, F_{i_3}, \cdots, F_{i_m},\}$ with $F_{i_1} \cap F_{i_2} \cap F_{i_3} \cap \cdots \cap F_{i_m}$.

(12 marks)

QUESTION FOUR (20 MARKS)

a) Let X be a topological space. Define what is meant by a set E is a component of X. (2 marks)

b) Consider the following topology on $X = \{a,b,c,d,e\}$.
$\tau = \{X, \Phi, \{a\}, \{d,e\}, \{a,c,d\}, \{b,c,d,e\}\}$. Find all the components of X. (6 marks)

c) Show that the subset $A = \{(x,y): x^2 - y^2 \le 4\}$ of $\Re$ is disconnected. (6 marks)

d) Show that every discrete space X is locally connected. (6 marks)

QUESTION FIVE (20 MARKS)

a) Let $A = (0,1)$, $B = (1,2)$ and $C = [2,3)$ be intervals on $\Re$. Show that A and B are separated but B and C are not. (5 marks)

b) Consider the following topology on $X = \{a,b,c,d,e\}$. $\tau = \{X, \Phi, \{a,b,c\}, \{c,d,e\}, \{c\}\}$.
Show that $A = \{a,d,e\}$ is disconnected. (3 marks)

c) Show that if A and B are non-empty separated sets, then $A \cup B$ is disconnected.

(4 marks)

d) Let A be a non-empty set. If $G \cup H$ is a disconnected of A, show that $A \cap G$ and $A \cap H$ are separated. (8 marks)