MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF EDUCATION SCIENCE

SMA 407: MEASURE THEORY

DATE: 8/8/2016 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

SECTION A: (COMPULSORY)

QUESTION ONE 30 MARKS

a) Define a measure? (3 marks)

b) Prove that if μ is a measure defined on a σ −algebra $\mathfrak{X}$. Then μ is monotonic, that is if $E \subset F$, then $\mu(E) \leq \mu(F)$. Furthermore if $\mu(E) \leq \infty$, then $\mu(F - E) = \mu(F) - \mu(E)$.
. (5 marks)

c) Prove that $\mu^*(\{x\}) = 0$ for all $x \in \mathbb{R}$ (4 marks)

d) Show that if $\mu^*(E) = 0$, then E is L-measurable. (5 marks)

e) Prove that the space $(\mathbb{R}, \mathcal{M}, \mu)$, where μ is the lebesque measure is complete. (5 marks)

f) Prove that if $f, g: \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and c be a real number then the function cf is $\mathfrak{X}$ −measurable (5 marks)

g) Prove that if φ and ρ are simple functions in $M^+(X, \mathfrak{X})$ and $c \geq 0$, then $\int c\varphi du = c \int \varphi du$ (3 marks)

QUESTION TWO 20 MARKS

a) Prove that if $f, g: \to \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and c be a real number then the function

i) f^2

ii) $|f|$ are $\mathfrak{X}$ −measurable. (5 marks)

b) Let (f_n) be a sequence of $\mathfrak{X}$ −measurable functions $f_n: X \to \mathbb{R}$, then the functions $f_1 \cup f_2 \cup f_{3\cup} \dots \dots \dots \cup f_n$ and $f_1 \cap f_2 \cap f_3 \cap \dots \dots \dots \cap f_n$ is $\mathfrak{X}$ −measurable. (5 marks)

QUESTION THREE 20 MARKS

a) State and prove monotone convergence theorem. (10 marks)

b) Let $(X, \mathfrak{X})$ be a measurable space. Then if $f, g \in M^+(X, \mathfrak{X})$, then

i) $f + g \in M^+(X, \mathfrak{X})$ (6 marks)

ii) $\int cf du = c \int f du$ (4 marks)

QUESTION FOUR 20 MARKS

a) Prove that if f and g both belong to M^+ and $f \leq g$, then $\int f du \leq \int g du$ (6 marks)

b) Prove that if $f \in M^+$ and if $E, F \in \mathfrak{X}$, with $E \complement F$ then $\int_E f du \leq \int_F f du$ (7 marks)

c) State and prove Fatous lemma. (7 marks)

QUESTION FIVE 20 MARKS

a) Define σ −algebra (5 marks)

b) Prove that if $A \complement B$ then $\mu^*(A) \leq \mu^*(B)$, $A, B \in \mathbb{R}$ (5 marks)

c) Prove that $\mu^*(\emptyset) = 0$ (3 marks)

d) Prove that lebesque outer measure μ^* is countably sub additive. (7 marks)