MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF EDUCATION SCIENCE

SMA 302: GROUP THEORY

DATE: 9/8/2016 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

Answer <u>ALL</u> the questions in Section A and <u>ANY TWO</u> Questions in Section B

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Define the following terms

i) Isomorphism

ii) Index of a subgroup.

iii) Order of a group

iv) A transposition

v) Relations (5 marks)

b) Construct the multiplication table for A_4 (5 marks)

c) Prove that the right cosets of H in G are one to one correspondence. (5 marks)

d) Define a group (5 marks)

e) Construct the addition table for Z_{36} and show that $\{0,5\}$ $and\{0,1\}$ are not the proper subgroups and construct its lattice diagram. (5 marks)

f) Prove that the group (G) of prime order is cyclic. (5 marks)

QUESTION TWO

a) Prove that If G is a group with binary operation * then the right and the left cancellation hold in G. (5 marks)

b) Prove that if G is a group and H non empty subset of G then H is a subgroup of G iff for $a, b \in H;\ ab^{-1} \in H.$ (5 marks)

c) Prove that the collection of all even permutation of a finite set of n elements of S_n forms a subgroup of order $\frac{n!}{2}$. (5 marks)

d) Prove that any two right (left) cosets of H are disjoint. (5 marks)

QUESTION THREE

a) State and prove the Lagranges' Theorem (6 marks)

b) Let H be a subgroup of G. then each coset has the same number of elements as H .
. (5 marks)

c) Define a function $f: G \to G^1$ by $G(x) = axa^{-1}$ show that f is isomorphism. (4 marks)

d) Find all the subgroups of D_3 and construct its lattice diagram (5 marks)

QUESTION FOUR

a) Define $f: R \to R^+$ by $(a) = e^a\ \forall a \in R$. Show that f is an isomorphism of $(R, +)$ onto $(R^+,\ .)$ (6 marks)

b) Let G denotes the set of all ordered pair of real numbers with non zero 1st component of the binary operation $*$. $*$ is defined on G by the rule $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G,*)$ is a group. (8 marks)

c) Let G be a group and let $a \in G$. The $H = \{a^n | n \in Z\}$ is a subgroup of G and is the smallest subgroup which contains a. proof (6 marks)

QUESTION FIVE

a) Let A be a non empty set and let S_A be the collection permutations of A. Then S_A is a group under permutation multiplication. Proof (5 marks)

b) Define * on Q^+ by $* b = \frac{ab}{2}$. Hence show that Q^+ with the binary operation $*$ is a group . (5 marks)

c) If G is a group with binary operation * and if a and b are any elements of G then the linear equation $y * a = b \; and \; x * a = b$ have a unique solution in G. proof. (5 marks)

d) Prove that the identity and the inverse of a group is unique (5 marks)