MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS

SMA 362: OPERATION RESEARCH

DATE: 8/8/2016 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer ALL the questions in Section A and ANY TWO Questions in Section B

QUESTION ONE 30 MARKS

a) Define the following terms as used in operation research.
   i) Objective function
   ii) Constraints
   iii) Mathematical model
   iv) Optimization problem
   v) Feasible solution (5 marks)

b) A soft drinks manufacturer has two machines A and B for bottling 200ml and jumbo sized bottles. The rate of bottling are

Machine	Number of 200ml bottles per min.	Number of jumbo bottles per min.
A	100	40
B	60	75

The machine can be run for 8 hours 5 days a week. The market demands for the two types of bottles are at most 25000 and 8000 per week and the profit are KSh.15 and Ksh.75 per bottle. Formulate this as a linear programming for determining number of bottles to be manufactured, of each type per week for maximum total profit. (5 marks)

c) Mr. Wafula has a 50 hectares piece of land. He wishes to plant tomatoes and onions. He has a capital of Ksh. 2700. One hectare of tomatoes cost Ksh. 60 to cultivate and onions cost Ksh. 30 to cultivate. He has a work force of 160 labourers and it takes 2 labourers to cultivate an hectare of tomatoes and 4 labourers to cultivate an hectare of onions. Suppose that he gets a profit of Ksh. 30 from tomatoes and 60 from onions

i) Set up a linear programming model (4 marks)

ii) Solve the problem using graphical method. (5 marks)

d) Solve using simplex method

$$Maximize\ z = 3x + 2y$$

$$s.t.\ \ 2x + y \leq 5$$

$$x + y \leq 3$$

$$x, y \geq 0$$ (6 marks)

e) A marketing manager has five salesmen A, B, C, D, E and five sales districts. Considering the capabilities of a salesman and the nature of the districts, the marketing manager estimates that the sales per month for each salesman in each district would be as follows;

	1	2	3	4	5
A	32	40	41	22	29
B	38	24	27	38	33
C	40	28	33	41	40
D	28	21	30	36	35
E	40	36	37	36	39

Find the assignment of salesman to district that will result in maximum sales. (5 marks)

QUESTION TWO 20 MARKS

a) A factory produces two products A and B with contribution of Ksh. 80 and Ksh. 10 per unit respectively. Product data are as below. (per unit)

Labour	Labour per Hour	Material X	Material Y
A	3	4	6
B	5	2	8
Total Available	500	350	500

i) Formulate the Linear programming model in a standard manner (4 marks)

ii) Solve the model using graphical method (6 marks)

iii) Solve using simplex method (6 marks)

iv) Calculate the shadow prices of the binding constraints and interpret. (4 marks)

QUESTION THREE 20 MARKS

a) Differentiate between surplus variables and slack variables. (2 marks)

b) Three products A,B, and C are produced in three machines centers X, Y and Z. each product involves operation of each of the machine centers. The time required for each operation on various products are indicated in the table below. 100, 77 and 80 hours are only available at machine center X,Y AND z respectively

	Machine centers			Profit
Product	X	Y	Z	
A	10	7	2	12
B	2	3	4	3
C	1	2	1	1
Available Hours	100	77	80	

i) Formulate a linear programming model (5 marks)

ii) Solve using simplex method (7 marks)

iii) Use simplex method to solve

$$\text{Min } Z = 4x + 5y + 7z$$
$$S.t. 6x + 8y + 10z \geq 150$$
$$x + 3y + 5z \geq 80$$
$$x, y, z \geq 0$$

(6 marks)

QUESTION FOUR 20 MARKS

a) A company has 4 fitters and has been asked to deal with 5 jobs. The times for each job are estimated as follows.

J O B S		Fitters			
		Kamau	Mwenda	Kioko	Steve
	1	6	12	20	12
	2	22	18	15	20
	3	12	16	18	15
	4	16	8	12	20
	5	18	14	10	17

Allocate the men to the jobs so as to minimize the total time taken and identify the job which will not be dealt with (10 marks)

b) The product of the two plants A and B are to be transported to three warehouses W1, W2, W3. The cost of transportation of each unit from the plant to the warehouses along with the normal capacities of the plant and warehouses are indicated below.

c)

Plants	Warehouses			
	W1	W2	W3	
A	25	17	25	300
B	15	10	18	500
	300	300	500	

Find the minimum cost solution for the transportation problem. (5 marks)

d) Find the optimal solution by solving the dual of the following primal problem

$$Minimize\ Z = 30a + 60b + 20c$$
$$S.t\ 5a + 10b + 15c \geq 200$$
$$2a + 3b + c \geq 200$$
$$8a + 6b + 4c \geq 650$$
$$a, b, c \geq 0$$

(5 marks)

QUESTION FIVE 20 MARKS

a) Define sensitivity analysis (2 marks)

b) Consider the following mathematical model

$$Max\ Z = 3x + 2y$$
$$S.t. x + y \leq 15$$
$$2x + y \leq 28$$
$$x + 2y \leq 20$$
$$x, y \geq 0$$

Suppose an extra 3 units are added to the available 15 units available. Find the optimal solution. (10 marks)

c) Find the optimal solution of

$$Max\ Z = 5x + 3y + 4z$$
$$S.t. 3x + 12y + 6y \leq 720$$
$$6x + 6y + 3z \leq 1260$$
$$6x + 9y + 9z \leq 1080$$
$$z \geq 20, \quad x, y \geq 0$$

(8 marks)