MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

SUPPLEMENTARY EXAMINATIONS

ECU 107: ENGINEERING MATHEMATICS IV

DATE: 1/8/2016 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (20 MARKS)

a) Evaluate $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ (3 marks)

b) Consider the function $f(x) = \begin{cases} \frac{x^2 - 1}{x - 2} & x \neq 2 \\ 0 & x = -2 \end{cases}$

i. Sketch the graph of the function $f(x)$ (2 marks)

ii. Determine the points of discontinuity(if any) of the function $f(x)$ (3 marks)

c) Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$ (3 marks)

d) A projectile is fired straight upwards with a velocity of $400m/s$; its distance a bove the ground t sec. after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired. Calculate the maximum height achieved by the particle (3 marks)

e) Evaluate $\int \sin(3x-1)dx$ (3 marks)

f) Determine Taylors series expansion generated by the function $f(x) = \ln x$ upto the fifth term about $x = 2$ (4 marks)

g) Calculate the volume obtained by rotating the area under the curve $y = 1 + x$ between $x = 1$ and $x = 2$ about the axis of x (4 marks)

h) Show that improper integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent (5 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^3 + x^2 - 5x - 5$

i) Use the second derivative test to find the local maximum and minimum of the function $f(x)$ (7 marks)

ii) Discuss the concavity of $f(x)$ (5 marks)

iii) Determine the points of inflection of $f(x)$ (4 marks)

iv) Sketch the graph of the function $f(x)$ (4 marks)

QUESTION THREE (20 MARKS)

a.) A vessel containing water is in the form of an inverted hollow cone with semi-vertical angle of 30^0. There is a small hole at the vertex of the cone and the water is running out at a rate of $3cm^3$ per second $(3cm^3/s)$. Calculate the rate at which the surface area in contact with water is changing when there is $81\pi\ cm^3$ of water remaining in the cone.

(6 marks)

b.) Evaluate $\int \frac{3x+7}{(x-1)(x^2+1)}dx$ (6 marks)

c.) Prove that $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ (5 marks)

d.) Calculate $\int e^{\sin x}\cos x dx$ (3 marks)

QUESTION FOUR (20 MARKS)

a.) Determine $\frac{dy}{dx}$ for;

i.) $y = \sinh x$ (2 marks)

ii.) $y = \tanh x$ (2 marks)

iii.) $y = \frac{(x^2+1)^5}{\sqrt{1+x}}$ (3 marks)

b.) Calculate $\frac{d^2y}{dx^2}$ if;

i.) $x = t - t^3 \quad ; \quad y = t - t^2$ (3 marks)

ii.) $x = \cos t \quad ; \quad y = (1-\sin^2 t)^{\frac{1}{2}}$ (3 marks)

c.) Calculate the gradient of the curve $x^3 + 2x^2y^2 - 2y + xy = 2$ at the point $(-4, 1)$

(7 marks)

QUESTION FIVE (20 MARKS)

a.) Evaluate

i.) $\lim_{x\to 0^-} \frac{|x|}{x}$ (3 marks)

ii.) $\lim_{x\to 2^+} \frac{(x^2+2)|x|}{x^2}$ (3 marks)

b.) Consider the function $f(x) = \begin{Bmatrix} 7x-2 & x \geq 2 \\ 3x+5 & x < 2 \end{Bmatrix}$

i.) Evaluate $\lim_{x\to 2^-} f(x)$ (3 marks) ii.) Sketch the graph of the function $f(x)$ (3 marks)

c.) Calculate the slope of the tangent line to the curve $x^3 + 2xy - 3y^2 = 9$ at the point $(3, 2)$

(4 marks)

d.) If $xy + y^2 = 1$ find;

i.) $\dfrac{dy}{dx}$ (2 marks)

ii.) $\dfrac{d^2 y}{dx^2}$ (2 marks)