MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER SUPPLEMENTARY EXAMINATIONS FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF ARTS IN MATHEMATICS AND ECONOMICS

SMA 360: MULTIVARIATE STATISTICAL METHODS 1

DATE: 7/8/2019 **TIME:**

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE (30 MARKS)

a) Define the following terms:

i. Mean Vector (1 mark)

ii. Random vector (1 mark)

b) List the three properties of Moment Generating Function (M.G.F). (3 marks)

c) Suppose that x_1, x_2, x_3 are random variables having a joint probability density function given by $f(x_1 x_2 x_3, x_4) = \begin{cases} 16x_1x_2x_3x_4, 0 \le x_i \le 1 \\ 0, elsewhere \end{cases}$. Determine the marginal distribution of:

i) $x_2 x_4$ (3 marks)

ii) x_2 (3 marks)

d) A three dimensional random vector x has a PDF given by $f(x_1 x_2 x_3) = \begin{cases} 5x_1x_2x_3, 0 \le x_i \le 1, i = 1, 2, 3 \\ o, elsewhere \end{cases}$. Determine $E(2x_1 + 4x_2^2 + 3x_3^3)$ (4 marks)

e) Suppose $x \sim N(0,1)$. Find the characteristic function x. (4 marks)

f) Suppose $x_1, x_2 \cdots x_n$ are independent Bernoulli random variables with parameter, θ. Determine the distribution of $y = \sum_{j=1}^{n} x_j$. (4 marks)

g) Let x_1, x_2, x_3 be independent standard normal random variable. Using the change of variable technique determine the probability distribution $\underset{\sim}{y} = [y_1 y_2 y_3]'$ where $y_1 = x_1, y_2 = x_1 + x_2$ and $y_3 = x_1 + x_2 + x_3$. (4 marks)

h) Show that $\bar{X}_n \xrightarrow{p} \mu$ using Chebyshev's inequality. (3 marks)

QUESTION TWO (20 MARKS)

a) Let x have the variance covariance matrix given by $\sum = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$. Find:

i. The correlation matrix of x (5 marks)

ii. The correlation coefficient between x_1 and $\frac{1}{3}x_2 + \frac{2}{3}x_3$ (4 marks)

b) Given the joint probability mass function of $x = [x_1, x_2, x_3]'$ as $f(x_1 x_2 x_3) = \begin{cases} kx_1 x_2 x_3, \; for 0 \le x_i \le 1, i = 1,2,3 \\ 0, otherwise \end{cases}$. Determine:

i. The value of k (5 marks)

ii. The marginal PDF of x_1 and x_2. (3 marks)

iii. The conditional distribution of $x_3 / x_1 = 1, x_2 = 2$ (3 marks)

QUESTION THREE (20 MARKS)

a) Briefly explain what is meant by quadratic form of $x_1, x_2 \cdots x_n$. Hence given that $x = [x_1, x_2, x_3]'$ has a mean vector and variance-covariance matrix given by $\mu = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\sum = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$ respectively. Find the expected value of the quadratic form $Q = (x_2 - x_3)^2 + (x_1 - x_2 + x_3)^2$ (10 marks)

b) Suppose $x=[x,y]'$ has a two variate normal distribution with density $f(x)=c\exp-\frac{1}{2}Q$.

Where $Q=\frac{1}{7200}(1600x^2+900xy+1125y^2+4140x+450y+5800)$. Identify $\mu,\sum$ and

(10 marks)

QUESTION FOUR (20 MARKS)

a) Let x_1,x_2,x_3 be independent and identically distributed random variables from a distribution whose probability density function is given by $f(x)=\begin{cases} e^{-x}\ for, x>0 \\ o, elsewhere \end{cases}$. Using the change of variable technique determine the joint distribution of $Y_1=\frac{x_1}{x_1+x_2+x_3}$ and $Y_2=\frac{x_1+x_2}{x_1+x_2+x_3}$. Hence find the marginal distribution of $Y_2/Y_1=y_1$. (6 marks)

b) Given $M(t_1t_2)=\left[(1-p_1-p_2)+p_1e^{t_1}+p_2e^{t_2}\right]^n$. Find:

i) $E(x_2)$ (2 marks)

ii) $E(x_1x_2)$ (2 marks)

iii) δ_{12} (4 marks)

iv) ℓ_{12} (1 mark)

c) Consider a random vector $x=[x_1x_2x_3]'$ defined as $x\sim N(\underset{\sim}{\mu},\sum)$ where $\underset{\sim}{\mu}=\begin{pmatrix}3\\4\\5\end{pmatrix}$ and

$\sum=\begin{bmatrix}11 & 5 & 2\\ 5 & 4 & 1\\ 2 & 1 & 3\end{bmatrix}$. Find the mean and variance of the conditional density of x_1 given $x_2=5$ and $x_3=5$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let Z_n be a chi-square with n degrees of freedom. Find the limits distribution of $y = \frac{z_n - n}{\sqrt{2n}}$. (10 marks)

b) Let $f(x)$ be a density function with mean μ and finite variance δ^2. Let $\overline{x_n}$ be the sample mean from $f(x)$ and let $\varepsilon > 0$ and $0 < \Delta < 1$ be any specified numbers.

i) If n is any integer greater than $\frac{\delta^2}{\varepsilon^2 \Delta}$ show that $p\left[-\varepsilon \leq \overline{x_n} - \mu \leq \varepsilon\right] > 1 - \Delta$. (5 marks)

ii) How large a sample should be taken in order that you are 99% certain that $\overline{x_n}$ is within 0.5δ of μ. (3 marks)

c) A fair die is tossed 12 independent times. Determine the probability of the following combination: (2 marks)

Face	1	2	3	4	5	6
Frequency	2	3	0	2	4	1