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Modeling Stock Returns Volatility Using Regime Switching Models

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Abstract

This paper seeks to model the dynamic relationship between stock market returns, volatility and trading volume in both developed and emerging stock markets. Modeling stock returns volatility has a tremendous reflection of the stock market microstructure behavior. We model this relationship using GARCH model, which previously has been used and reproduced most stylized facts of financial time series data, and compare its results with those of Regime-Switching and Markov-Switching GARCH. The results indicate evidence of volatility clustering, leverage effects and leptokurtic distribution for the index returns. Moreover, we find that all the three stock markets are characterized by return series process staying in low volatility regime for a long time than in high volatility regime. Markov-Switching GARCH (1, 1) model is reported to be a better model than GARCH (1, 1).

Keywords: Volatility, Stock Returns, GARCH, Regime Switching, Markov-Switching GARCH

INTRODUCTION

In the previous financial modeling studies, the connection between stock returns, volatility and volume has been widely examined in both developed and emerging stock markets. This relationship is paramount since it provides insights into the understanding of the microstructure of financial markets. Volatility can be defined as a statistical measure of dispersion of returns for a given security or market index which can be measured by either using the standard deviation or variance of the return series from a given market index. A higher value of standard deviation implies a greater dispersion of returns and high risk associated with the investment. The performance of stock market has strong dependence on volatility and thus as volatility declines the stock market rises and vice versa. This means that when volatility increases, risk increases and returns decrease. On the other hand, Abbontante [1] defines trading volume as the total number of shares traded each day. Trading volume can be used to measure the value of stock price movement. That is, trading volume is an important technical indicator that is used to confirm a trend or a trend reversal. It gives investors idea of the price action of a security and whether they should buy or sell the security. If trading volume increases prices generally move in the same direction. We therefore note that the understanding of connection between stock returns, volatility and volume should be given a considerable attention in financial modeling. A study by Wiley and Daigler [17] contend that the relationship between price and volume is attributed to the role of information in the price formation. On the other hand, Karpoff [11] argue that price-volume empirical relationship is paramount because of the fact that it gives insights in the understanding of the many theories that compete to widely spread ideas about information flow into the market.

Moreover, Attari et.al [3] in their research uncovers that investors are encouraged by higher returns to invest and consequently there is capital inflow while in the volatile environments the returns are not certain and it is difficult to predict the investment. The study of Attari et.al [3] and Hseih [10] reveals that emerging stock markets associates with highly volatile stock return due to low stock market volume. Girard and Biswas [8] in their study on trading volume and stock return volatility in developed versus emerging stock markets reports negative relation between the two. According to Al-Samman and Al-Jafari [2], trading volume is considered as an important technical indicator in measuring the strength of a market for the reason that it contains about stock behaviour. The contemporaneous and dynamic relationship between trading volume and stock returns has been a cause for empirical studies. Lee and Rui [13] revealed that returns granger cause trading volume in the developed markets (US, UK and Japan). In their study, De Medeiros and Doornik [7] report a contemporaneous and dynamic relationship between returns, volatility and trading volume by using Brazilian market data. A positive correlation between trading volume and volume was reported by Mahajan and Singhn [14]. Moreover, studies by Christie [5] reveal a negative correlation between volume and volatility. Crouch [6] studied the connection between daily trading volume, absolute changes of the stock market and individual stocks and found a positive relationship between them. Rogalski [16] utilized month to month stock information and found a positive contemporaneous relationship between trading volume and absolute returns.

More recently, a look into by Komain [12] aimed to inspect the dynamic connection between stock return, trading volume, and volatility in the Thai Stock market reports that trading volume assumes a prevailing job in the dynamic connections. Specifically, trading volume causes both return and return volatility when the US subprime emergencies are considered. More generally, literature reveals that there is contemporaneous connection between trading volume and return in the Thai securities exchange.

In general, despite the significant efforts that have been made, empirically and theoretically, on the phenomenon of stock returns, volatility and volume relationship on different stock markets, mixed results have been reported. However, most of the findings have confirmed existence of contemporaneous relationship between stock returns, volatility and trading volume. We note that, despite a fair amount of empirical evidence that exists on the connection between stock returns, volatility and trading volume for developed and emerging financial markets, there exists few studies that have investigated the case of Nairobi Securities Exchange (NSE) (an emerging market) compared to a developed stock market. Our study therefore seeks to investigate the kind of relationship between stock returns, volatility and trading volume in both emerging and developed financial markets by applying GARCH and Regime Switching models.

The rest of this paper is organized as follows; Section 2 provides an overview of the GARCH, Regime Switching models and Markov-Switching GARCH models used in the study. Section 3 presents the data used and its descriptive statistics and the general discussion of the study findings. Finally, section 4 concludes the paper.

METHODOLOGY

GARCH Models

The generalized autoregressive conditional heteroscedasticity (GARCH) models, which have been widely used in financial and economic modeling and analysis, are characterized by their ability to capture volatility clustering among other stylized facts and to account for non-uniform variance in time-series data. Bollerslev [4] proposed these models as a valuable extension of ARCH model. By defining a log return series r_t, we assume that the mean equation of the process can be sufficiently described by an ARMA model. Let $a_t = r_t - \mu_t$ be the mean-corrected log return. Then a_t follows a GARCH (p, q) model if

$$a_t = \alpha_t \varepsilon_t \ , \qquad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad [1]$$

where q is the degree of GARCH; p is the degree of the ARCH process, ε_t is a sequence of i.i.d random variable with mean 0 and variance 1, $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_j) < 1$ We note that $\alpha_i = 0$ for $i > p$, and $\beta_j = 0$ for $j > q$ and the constraint $\alpha_i + \beta_j < 1$ imply that the unconditional variance of a_t is finite, whereas its conditional variance a_t^2 evolves over time. Equation [1] reduces to a pure ARCH (p) model if $q = 0$. GARCH (1, 1) is the basic and most widespread model and it is expressed as

$$a_t = \alpha_t \varepsilon_t , \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad [2]$$

In this model, α_0 is the non-changing variance that corresponds to the long run average, α_1 represents the first order ARCH term which broadcasts information pertaining volatility from an earlier period, and β_1 is the first order GARCH term which is the new information that was not available at the time the previous forecast was made. If $\alpha_1 + \beta_1 > 1$, it implies that the shocks to volatility do not die off over time. This is to mean that the magnitude of these parameters determines the extent of volatility persistence. The closer the sum of α_1 and β_1 to 1, the more the shocks to volatility does not die off.

Regime Switching Models

The Regime Switching model was developed by Hamilton [9] and is also referred to as the Markov Switching model. It is among the most popular nonlinear time series models in literature that have been used to model stock data. It involves multiple structures that characterize the time series behaviors in different regimes. The model is able to capture more complex dynamic patterns if switching between these structures is permitted as well as it allows coefficients of the conditional mean and variance to vary according to some finite-valued stochastic process with states or regimes $S_{t,} S_t \in S = (1, \dots \dots . . r)$.The regime changes aim at capturing changes in the underlying financial and economic mechanism through the observed time period.

We presume that a return r_t follows a simple two-state Markov switching model with different risk premiums and different GARCH dynamics expressed as

$$r_t = \begin{cases} \alpha_{10} + \alpha_{11} h_{t-1} + e_t, \;\; e_t \sim iid\ N(0, \sigma^2),\ if\ S = 1 \\ \alpha_{20} + \alpha_{21} h_{t-1} + e_t, \;\; e_t \sim iid\ N(0, \sigma^2), if\ S = 2 \end{cases} \qquad [3]$$

where h_{t-1} is the stock market return at time $t-1$, α_{10} and α_{11} are the parameter estimates in the first regime and α_{20} and α_{21} are the parameters in the second regime. The probability of transition from one state to another is governed by

$$P(S_t = 2\ / S_{t-1} = 1) = \ q_1\ , P(S_t = 1\ / S_{t-1} = 2) = \ q_2 \qquad [4]$$

where $0 < q_i < 1$. A small q_i means that the return series has a tendency to stay in the i^{th} state with expected duration $\frac{1}{q_i}$.

Markov-Switching GARCH Models

We shall note our variable of interest at time t by r_t.We assume that r_t has zero mean and has no serial correlation, that is, we assume the following moment conditions: $E\left[r_t\right] = 0$ and $E\left[r_t, r_{t-1}\right] = 0$ for $i \neq 0$ and all $t > 0$. According to McNeil et.al [15], this assumption is realistic for high frequency returns for which the (conditional) mean is often assumed to be zero. We allow for regime-switching in the conditional variance process. Denote by I_{t-1} the information set observed up to time $t-1$, that is, $I_{t-1} = [r_t, i > 0]$.The general Markov-Switching GARCH specification is expressed as: $r_t \big| (s_{t,}\ I_{t-1}) \sim D\big(0, h_{k,t}\big), \in_t$ where $D\big(0, h_{k,t}\big)$, $\in_t$ is a continuous distribution with mean zero, time-varying variance $h_{k,t}$ and additional shape parameters gathered in the vector $\in_t$. The integer-valued stochastic variable S_t , defined on the discrete space$1,2, \dots \dots, k$, characterizes the Markov-swiching GARCH model. We define the standardized innovations as $\eta_{k,t} = (r_t / h_{k,t})^{1/2} \sim D(0,1, \in_k)$. According to Bollerslev [4], the GARCH model is given by

$$h_{k,t} = \alpha_{0,k} + \alpha_{1,k} r_{t-1}^2 + \beta_k h_{k,t-1} \qquad [5]$$

for $k = 1, \dots . . K$. In this case, we have $\theta_k = (\alpha_{0,k}, \alpha_{1,k}, \beta_k)^T$. In order to ensure non-negativity (positivity), we demand that $\alpha_{0,k} > 0, \alpha_{1,k} \geq 0$ and $\beta_k \geq 0$.Covariance-stationarity in each regime is obtained by requiring $(\alpha_{1,k} + \beta_k) < 1$.

RESULTS AND DISCUSSION

Empirical Data

In our analysis we focus on the daily, weekly and monthly closing prices and trading volume as reported in FTSE100 index in London, S&P500 index in New-York and NSE20 share index in Nairobi. All the returns and volume are calculated as the first difference of the log of the daily, weekly and monthly closing price and volume.

Descriptive statistics

The basic statistical properties of the data are presented in table 1. The mean index returns are positive for all the index return series and close to zero. The positive mean of index returns is an indication that investors in these markets have earned a positive rate of return on their investment. The mean of volume for all series is positive except for the FTSE100 daily and NSE20 weekly indices. We observe that the mean of returns and volume are close to zero, however, as the data changes from daily to monthly, the means slightly increases. We further report that the standard deviations of monthly return series are higher in most of the time series than those for weekly and daily return series. There is negative skewness in most of the time series except for the NSE20 daily and weekly index returns and S&P500 and NSE20 weekly index volume which implies that the distribution of all the return series except NSE20 daily and weekly index returns and S&P500 and NSE20 weekly volume have long tail to the left. The distribution of all the return series and volume is leptokurtic as implied by the positive values of the kurtosis coefficients. Most return series have heavy tails as exhibited by the skewness and kurtosis coefficients which are statistically different from those of normal distribution which are 0 and 3 respectively. The kurtosis and the Jarque-Bera (JB) statistic decrease as the time scale increases from daily to monthly observations. The assumption of normality is clearly rejected by the values for JB statistic and this indicates presence of Heteroscedasticity.

Table 1: Descriptive statistics for daily, weekly and monthly stock returns and volume

Market	Time Series	Obs	Nobs	Mean	Std Dev	Skewness	Ex.Kurt	J-B test
FTSE100 index	Returns	D	4348	5.00e-4	0.012	-0.159	6.757	83000
		W	886	2.43e-4	0.024	-1.111	12.556	6035.2
		M	203	9.83e-4	0.040	-0.715	0.889	24.84
	Volume	D	4348	-2.1e-4	0.351	-0.011	13.418	32,652
		W	886	1.1e-4	0.025	-1.183	12.908	3535
		M	203	3.9e-4	0.042	-0.695	0.715	1,299
S&P500 index	Returns	D	4275	1.72e-4	0.012	-0.220	9.389	15757
		W	918	8.43e-4	0.024	-0.928	8.091	2651.3
		M	203	3.31e-3	0.042	-0.861	1.888	57.16
	Volume	D	4275	1.03e-3	0.782	0.093	2.224	14,507

		W	918	3.92e-3	0.880	-0.007	1.601	2341
		M	203	0.01033	1.036	-0.232	1.286	2.05
NSE20 index	Returns	D	4271	1.56e-4	0.009	0.393	11.322	22949
		W	879	7.56e-4	0.026	0.418	5.872	1296.9
		M	200	3.36e-3	0.060	-0.467	2.185	49.06
	Volume	D	4271	1.8e-4	0.1891	-0.096	9.017	888.1
		W	879	-1.1e-3	0.2511	0.093	7.797	95.08
		M	200	4.2e-3	0.1600	-0.039	0.452	16.54

D=daily, W=weekly, M=monthly observations and Nobs=Number of observations

Empirical Findings and Discussion

Figure 1 to 3 presents a time series plot of the FTSE100, S&P500 and NSE20 adjusted closing prices and volume for the three frequencies (daily, weekly and monthly observations). The plots reports clearly that the series have trend which is an indication that the means and variances of the time series are non-constant and therefore we can conclude that the data is non-stationary. Moreover, volatility clustering, that is, low volatility is followed by low volatility and high volatility is followed by high volatility, is evidently implied by the plots of index returns volatilities in Figures 4 to 6 for the three frequencies.

We estimate GARCH (1, 1) model assuming normality. Table 2 presents the results for parameter estimates for the daily, weekly and monthly index returns series. The condition $\alpha_1 + \beta_1 < 1$ and $\alpha_0 > 0, \alpha_1 > 0 \; and \; \beta_1 > 0$, which is significant for mean reverting (volatility persistence) process is met by the results. This implies that conditional volatilities are mean reverting for all the time series and frequencies and that the GARCH (1, 1) model is weakly stationary. Further, we note that, in most of the cases monthly conditional volatilities of index returns tend to revert quickly towards the mean as compared to weekly and daily conditional volatilities. Moreover, the first three coefficients α_0 (constant), ARCH term (α_1) and GARCH term (β_1) are highly significant for the three indices returns for all time scale observations with the exception that α_0 is not significant for monthly observations of FTSE100,S&P500 and NSE20 indices and μ is not significant for FTSE100(monthly) and NSE20(daily, weekly and monthly) indices. The significance of these parameters is an indication that lagged conditional variance and squared disturbance has an impact on conditional variance which means volatility news from the previous periods has the power to explain current volatility.

Table 4 to 6 presents the parameter estimates and transition probabilities for Regime Switching model for FTSE100, S&P500 and NSE20 index returns. As for the case of daily index returns, the parameter estimates are significant in both regimes except for FTSE100 in regime one which is not significant whereas in weekly index returns, only the parameter estimates for FTSE100 index are significant and all the parameters in S&P500 and NSE20 stock markets are not significant. Lastly, the results for monthly index returns indicate that the parameters are only significant in regime one for FTSE100 and S&P500. We also note that the probability that return series will transit from regime 1 to 2 is very low. However, once in one regime the process tends to remain there for a long time. The probability of being in regime 1 is higher than that of staying in regime 2 for all the daily time series whereas for weekly return series regime 1 has the highest probability of staying in regime 1 than in regime 2 for FTSE100 and S&P500 index returns. as for monthly return series, only FTSE100 has the highest volatility of staying in regime 1 than in regime 2. This means that the return series process for emerging market stays in high volatility regime for long than in low volatility regime contrary to developed stock markets.

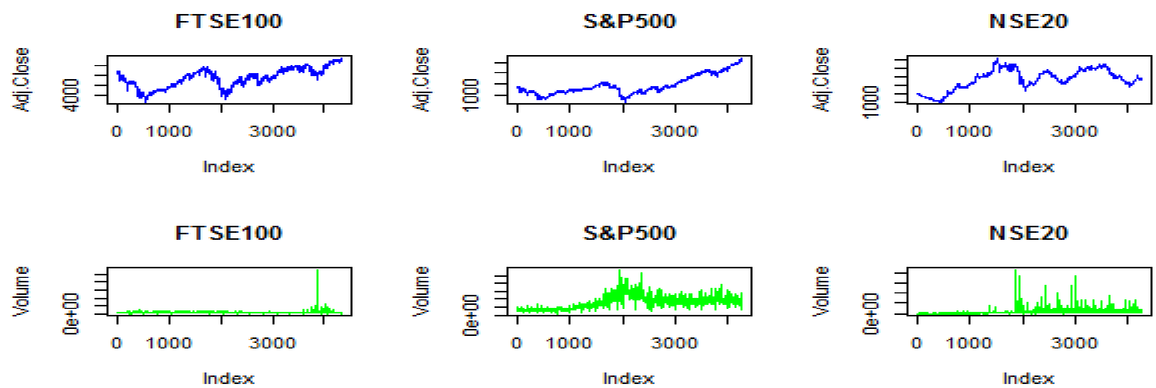

FIGURE 1: DAILY INDEX OBSERVATIONS AND VOLUME

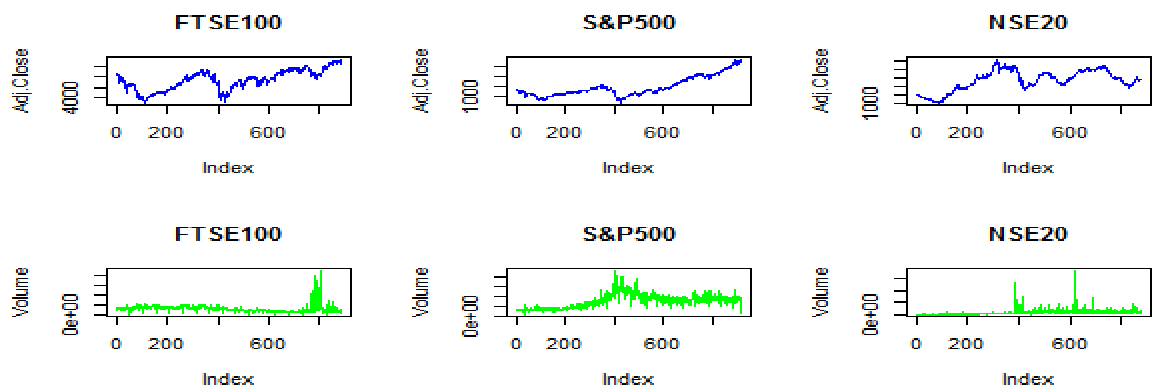

FIGURE 2: WEEKLY INDEX OBSERVATIONS AND VOLUME

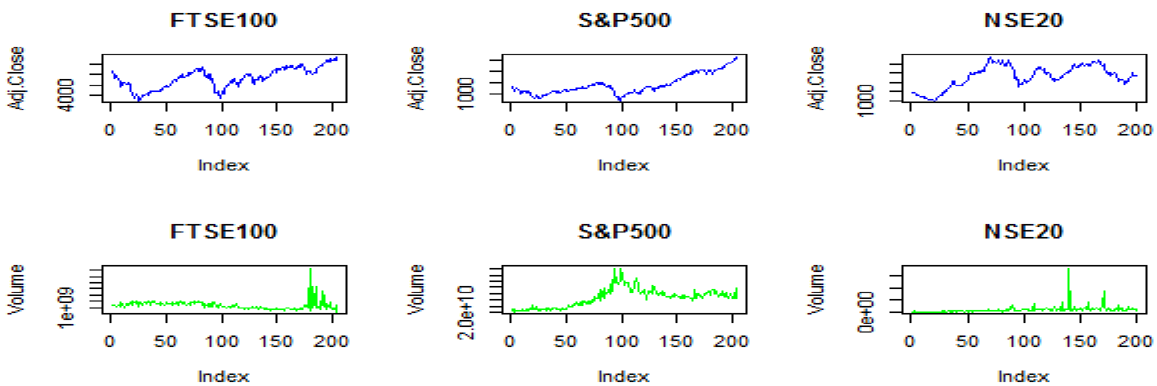

FIGURE 3: MONTHLY INDEX OBSERVATIONS AND VOLUME

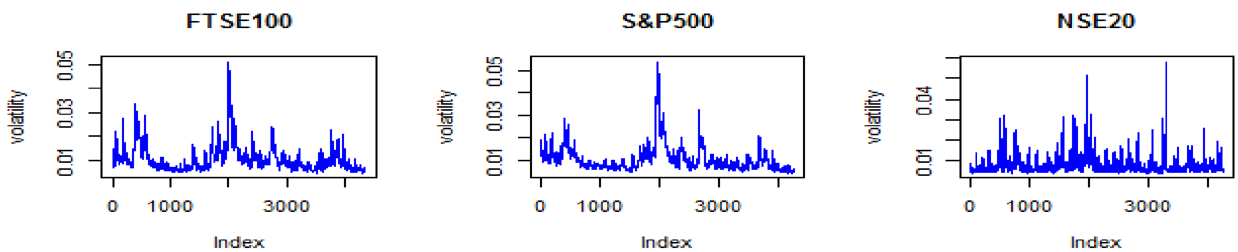

FIGURE 4: DAILY FTSE100, S&P500 AND NSE20 INDEX RETURNS VOLATILITY

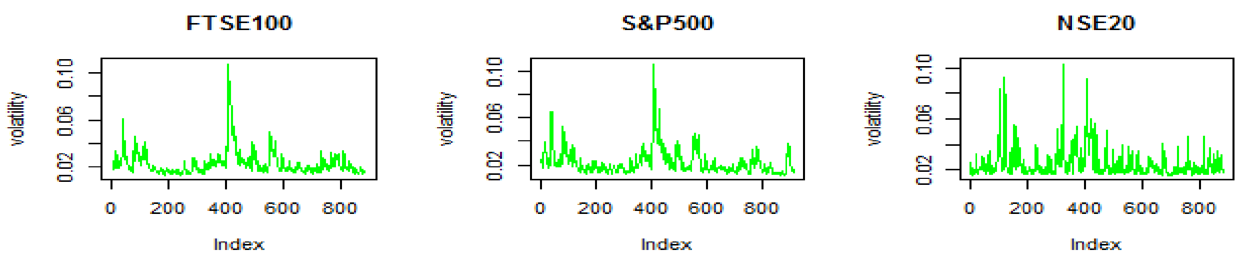

FIGURE 5: WEEKLY FTSE100, S&P500 AND NSE20 INDEX RETURNS VOLATILITY

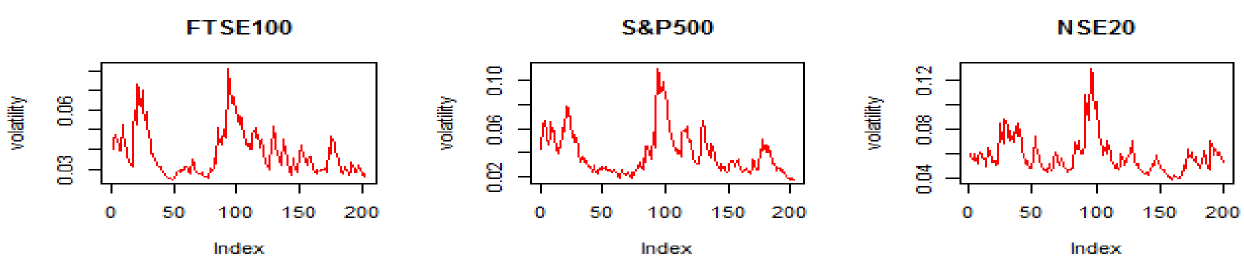

FIGURE 6: MONTHLY FTSE100, S&P500 AND NSE20 INDEX RETURNS VOLATILITY

Table 2: GARCH (1, 1) for returns

	FTSE100 index returns			S&P500 index returns			NSE20 index returns		
Parameter	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly
μ	3.811e-4 0.001**	1.470e-3 0.016 *	4.242e-3 0.066	5.254e-4 9.6e-6***	2.371e-3 1.6e-5***	7.90e-3 8.5e-5***	1.488e-4 0.136	4.046e-4 0.595	0.003 0.406
α_0	1.661e-6 2.6e-7***	2.911e-5 0.005 **	1.224e-4 0.125	1.627e-6 4.35e-9 ***	2.781e-5 0.001***	5.79e-5 0.167	4.607e-6 4e-15 ***	1.30e-4 2.9e-6***	0.0003 0.145
α_1	1.102e-1 <2e-16***	1.907e-1 1.5e-6 ***	1.826e-1 0.007**	9.749e-2 <2e-16***	2.240e-1 1.2e-9***	2.45e-1 1.0e-3***	2.68e-1 <2e-16***	3.44e-1 2.4e-7***	0.157 0.049*
β_1	8.778e-1 <2e-16***	7.738e-1 <2e-16***	7.396e-1 1.1e-15***	8.883e-1 <2e-16***	7.379e-1 <2e-16***	7.32e-1 <2e-16***	6.92e-1 <2e-16***	4.833e-1 9 e-10***	0.755 8.3e-13***
$\alpha_1 + \beta_1$	0.988	0.9645	0.9222	0.98579	0.9619	0.977	0.960	0.8273	0.912
Logl	14077	2152	382	13928	2280	385	15066	2072	287
JB	191.3 0***	724.1 0***	16.18 0.000***	569.2 0***	125.56 0***	12.30 0.002***	7089 0***	274.64 0***	9.711 0.008***
Q(10)	5.556 0.851	9.204 0.513	5.030 0.889	23.675 0.009	6.194 0.799	6.599 0.763	527.81 0***	67.78 1.19e-10	26.928 0.003
Q(15)	11.872 0.689	15.24 0.434	8.217 0.915	32.227 0.006	7.653 0.937	8.551 0.900	547 0***	81.68 3.435	29.50 0.014
Q(20)	15.478 0.748	19.792 0.471	15.994 0.717	38.362 0.008	13.372 0.861	25.719 0.175	560.381 0***	94.141 1.39e-11	34.791 0.021
LM test	12.017 0.444	15.520 0.214	8.575 0.739	17.279 0.139	11.675 0.472	8.270 0.764	10.585 0.565	12.189 0.431	17.100 0.146
AIC	-6.473	-4.850	-3.731	-6.514	-4.959	-3.755	-7.053	-4.707	-2.840
BIC	-6.467	-4.828	-3.665	-6.508	-4.938	-3.689	-7.047	-4.685	-2.774

Note: *, **, and *** indicate significance at 5%, 1% and 0.1% respectively.

Q (10), Q (15) and Q (20) are Ljung-Box tests at lag 10, 15 and 20 respectively

Logl is log likelihood, JB is Jarque-Bera test, AIC is the Akaike Information Criterion, BIC is the Bayesian Information Criterion

Table 3: GARCH (1, 1) for Trading volume

	FTSE100 Index Volume			S&P500 Index Volume			NSE20 Index Volume		
Parameter	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly
μ	-0.0021 0.577	-0.0075 0.404	-0.005166 0.73976	-4.3e-4 0.95802	-0.0108 0.129	-4.3e-4 0.95802	8.5e-3 0.867	0.01462 0.52937	8.455e-3 0.867
α_0	0.0070 9.8e-12**	0.0369 <2e-16***	0.0024 0.3221	0.0129 3.7e-5***	0.0322 1.2e-6***	0.0129 3.7e-5***	5.73e-1 2e-16***	0.0446 0.1109	5.7e-1 2e-16***
α_1	0.1820 <2e-16***	0.5077 1e-12***	0.2442 0.002**	0.4259 0.0015**	0.2832 7.1e-6***	0.42605 0.0015**	0.4036 5.4e-5***	0.1467 0.0053**	5.0e-1 5.4e-5***
β_1	0.7715 <2e-16***	0.3939 <2e-16***	0.7953 <2e-16***	0.0823 0.4886	0.2030 0.124	0.0823 0.4886	1e-8 -	0.8034 <2e-16***	1e-8 -
$\alpha_1 + \beta_1$	0.9535	0.3939	1.0395	0.5082	0.4862	0.5084	0.4036	0.9501	0.500
Logl	-943.9	-321	-43.679	97.629	25.84	97.63	-275.2	-1081	-275
JB	7295 0***	8707.2 0 ***	291.67 0***	0.0045 0.997	897.82 0***	0.0045 0.9977	1.996 0.3686	213.5 0***	1.996 0.3686
Q(10)	545.2 0 ***	52.15 1.07e-7***	61.35 2e-9***	63.53 7.7e-10***	113.98 0 ***	63.52 7.7e-10***	46.59 1.12e-6***	142.4 0 ***	46.59 1.1e-6***
Q(15)	670.18 0***	82.428 2.5e-11***	106.19 8.9e-16***	111.0 1.1e-16***	123.95 0***	11 5 1.1e-16***	53.47 3.2e-6***	152.88 0 ***	53.47 3.22e-6***
Q(20)	781.3138 0 ***	98.48 2.4e-12***	141.32 0***	127.74 0***	144.87 0 ***	127.74 0 ***	55.14 3.9e-5***	165.9 0 ***	55.14 3.9e-5***
LM test	28.823 0.0041***	2.4947 0.9981	12.483 0.4077	6.614 0.882	17.06 0.147	6.614 0.882	17.533 0.1306	11.66 0.4734	17.533 0.1306
AIC	0.4360	0.7336	0.4698	-0.9224	-0.0476	-0.9225	2.7920	2.470	2.792
BIC	0.4419	0.755	0.5351	-0.8572	-0.0266	-0.8572	2.8580	2.4918	2.8580

Note: *, **, and *** indicate significance at 5%, 1% and 0.1% respectively.

Q (10), Q (15) and Q (20) are Ljung-Box tests at lag 10, 15 and 20 respectively

Logl is log likelihood, JB is Jarque-Bera test, AIC is the Akaike Information Criterion, BIC is the Bayesian Information Criterion

Table 4: Regime switching model FTSE100, S&P500 and NSE20 daily index returns

		FTSE100		S&P500		NSE20	
Regime		Estimate	P-value	Estimate	P-value	Estimate	P-value
1	α_{10}	0.0005	5.733e-07***	0.0007	2.56e-12 ***	0.0001	0.3173
	α_{11}	-0.0089	0.6579	-0.0524	0.005817 **	0.2905	<2e-16 ***
2	α_{20}	-0.0010	0.04550 *	-0.0012	0.0455003 *	0.0003	0.6682
	α_{21}	-0.0579	0.04219 *	-0.0981	0.0006034***	0.3742	<2e-16 ***
		R1	R 2	R 1	R 2	R 1	R 2
	R1	0.9902	0.0232	R 1 0.9903	0.0232	R 1 0.9689	0.1478
	R2	0.0098	0.9768	R 2 0.0097	0.9768	R2 0.0311	0.8522

Table 5: Regime switching model FTSE100, S&P500 and NSE20 Weekly index returns

		FTSE100		S&P500		NSE20	
Regime		Estimate	P-value	Estimate	P-value	Estimate	P-value
1	α_{10}	0.0035	5.434e-09 ***	0.0007	0.8155	-0.0070	0.05185
	α_{11}	-0.1217	0.005046 **	0.0852	0.2380	-0.1052	0.18299
2	α_{20}	0.0021	0.0027 **	-0.0045	0.05041	0.0006	0.3173
	α_{21}	-0.0707	0.1121	-0.0486	0.42255	0.2123	2.15e-06***
		R 1	R 2	R 1	R2	R 1	R2
	R1	0.8874	0.0302	R 1 0.9793	0.0430	R 1 0.8113	0.0643
	R 2	0.1126	0.9698	R 2 0.0207	0.9570	R2 0.1887	0.9357

Table 6: Regime switching model for FTSE100, S&P500 and NSE20 Monthly index returns

		FTSE100		S&P500		NSE20	
Regime		Estimate	P-value	Estimate	P-value	Estimate	P-value
1	α_{10}	0.0125	1.906e-07 ***	0.0134	2.36e-08 ***	-0.0050	0.5215
	α_{11}	-0.3922	9.354e-06***	-0.2174	0.02938 *	1.4178	0.1946
2	α_{20}	-0.0100	0.1356	-0.0033	0.56266	0.0043	0.1112
	α_{21}	0.0772	0.5072	0.1883	0.06279	0.0396	0.5348
		R 1	R 2	R 1	R 2	R 1	R 2
	R 1	0.9476	0.0679	R 1 0.9665	0.0279	R 1 0.0169	0.1997
	R 2	0.0524	0.9321	R 2 0.0335	0.9723	R 2 0.9831	0.8003

Tables 7, 8 and 9 presents the MSGARCH (1, 1) parameters estimates for FTSE100,S&P500 and NSE20 daily, weekly and monthly index returns. We note that all the parameters are statistically significant an they signify that the volatility process evolution is heterogeneous across the two regimes. In fact, we note that the two regimes report different unconditional volatility levels as well as different volatility persistence. For instance, the first regime for daily FTSE100, S&P500 and NSE20 indices reports $\alpha_{1,1}+\beta_1 \approx 0.9914$, $\alpha_{1,1}+\beta_1 \approx 0.9815$ and $\alpha_{1,1}+\beta_1 \approx 0.9536$ while the second regime reports $\alpha_{2,2}+\beta_2 \approx 0.9662$, $\alpha_{1,1}+\beta_1 \approx 0.9959$ and $\alpha_{1,1}+\beta_1 \approx 0.6492$ respectively. The volatility persistence is generally high in the second regime for the three indices except in the FTSE100 (daily and monthly) and NSE20 (daily) indices returns. The significance of this property is that the first regime is characterized by low unconditional volatility and low persistence of the volatility process while the second regime is characterized by high unconditional volatility and high persistence of the volatility process. Evidently, regime one would be perceived as "steady market conditions" with low volatility levels and while regime two as "unstable market conditions" with high volatility levels and strong persistence. This is also confirmed by the transition probabilities which reports that process has the tendency to spend more time in regime one (low volatility).

The Markov chain transition probabilities indicate how the return series process moves from regime 1(low volatility) to regime 2 (high volatility). We notice that the probabilities of transiting from regime 1 to regime 2 are very low and consequently the probabilities of staying at the same state are very high. This means that when the process is at one regime, the probability of changing regime is very low. In general, all the probabilities spend more time at regime 1 than in state 2 except for FTSE100 and NSE20 weekly index returns and S&P500 and NSE20 monthly index returns which reports contrary results.

Table 7: MSGARCH (1, 1) model for FTSE100, S&P500 and NSE20 Daily index returns

Parameter	FTSE100		S&P500		NSE20	
	Estimate	P-value	Estimate	P-value	Estimate	P-value
$\alpha_{0,1}$	0.0000	<1e-16	0.0000	<1e-16	0.0000	3.360e-03
$\alpha_{1,1}$	0.0332	<1e-16	0.0678	<1e-16	0.0070	1.145e-01
β_1	0.9582	<1e-16	0.9137	<1e-16	0.9466	<1e-16
xi_1	0.9232	<1e-16	0.9084	<1e-16	0.9960	<1e-16
$\alpha_{2,1}$	0.0000	<1e-16	0.0000	<1e-16	0.0001	3.127e-04
$\alpha_{2,2}$	0.2433	<1e-16	0.2268	<1e-16	0.2896	7.236e-03
β_2	0.7229	<1e-16	0.7691	<1e-16	0.3596	3.235e-03
xi_2	0.7937	<1e-16	0.7683	<1e-16	1.0403	<1e-16
$\alpha_{11}+\beta_1$	0.9914		0.9815		0.9536	
$\alpha_{2,2}+\beta_2$	0.9662		0.9959		0.6492	
Transition probabilities	R1 / R2		R1 / R2		R1 / R2	
	R1 0.9839	0.0161	R1 0.7275	0.2725	R1 0.9625	0.0375
	R2 0.0367	0.9633	R2 0.9737	0.0263	R2 0.1254	0.8746
Logl	14110.6898		14013.3058		15270.0962	
AIC	-28201.3795		-28006.6115		-30520.1925	
BIC	-28137.6048		-27943.0061		-30456.5964	

Table 8: MSGARCH (1, 1) model for FTSE100, S&P500 and NSE20 weekly index returns

par	FTSE100		S&P500		NSE20	
	Estimate	P-value	Estimate	P-value	Estimate	P-value

$\alpha_{0,1}$	0.0000	<1e-16	0.0000	3.203e-01	0.0001	4.749e-02
$\alpha_{1,1}$	0.0875	<1e-16	0.0318	4.391e-02	0.0861	2.607e-01
β_1	0.8466	<1e-16	0.9372	<1e-16	0.6108	2.702e-05
xi_1	0.8206	<1e-16	0.9973	<1e-16	1.0041	<1e-16
$\alpha_{2,1}$	0.0021	<1e-16	0.0001	3.621e-03	0.0002	1.771e-01
$\alpha_{2,2}$	0.8883	<1e-16	0.3239	1.406e-01	0.1671	2.904e-01
β_2	0.1113	<1e-16	0.6565	<1e-16	0.8199	<1e-16
xi_2	0.0664	<1e-16	0.6510	<1e-16	1.0352	<1e-16
$\alpha_{1,1}+\beta_1$	0.9341		0.969		0.6969	
$\alpha_{2,2}+\beta_2$	0.9996		0.9804		0.987	
Transition probabilities	R1	R2	R1	R2	R1	R2
	R1 0.9525	0.0475	R1 0.3341	0.6659	R1 0.8795	0.1205
	R2 0.9999	0.0001	R2 0.6192	0.3808	R2 0.3221	0.6779
Logl	2194.9417		2302.9038		2118.0534	
AIC	-4369.8834		-4585.8075		-4216.1068	
BIC	-4322.0162		-4537.5855		-4168.319	

Table 9: MSGARCH (1, 1) model for FTSE100, S&P500 and NSE20 Monthly index returns

par	FTSE100		S&P500		NSE20	
	Estimate	P-value	Estimate	P-value	Estimate	P-value
$\alpha_{0,1}$	0.0000	3.492e-01	0.0001	1.783e-01	0.0015	2.254e-08
$\alpha_{1,1}$	0.0193	3.374e-01	0.2031	2.153e-01	0.0000	4.949e-01
β_1	0.9591	<1e-16	0.7470	<1e-16	0.0046	7.940e-02
xi_1	0.8495	2.118e-13	0.6694	1.041e-08	0.6517	2.737e-06
$\alpha_{2,1}$	0.0001	9.826e-03	0.0001	2.341e-01	0.0002	3.043e-02
$\alpha_{2,2}$	0.2226	3.788e-02	0.2030	2.643e-01	0.0675	3.470e-02
β_2	0.7549	<1e-16	0.7473	<1e-16	0.9305	<1e-16
xi_2	0.3972	3.172e-11	0.6694	3.615e-06	0.3369	3.357e-07
$\alpha_{1,1}+\beta_1$	0.9784		0.9501		0.0046	
$\alpha_{2,2}+\beta_2$	0.9775		0.9503		0.998	
Transition probabilities	R1	R2	R1	R2	R1	R2
	R1 0.9930	0.0070	R1 0.9821	0.0179	R1 0.8579	0.1421
	R2 0.0062	0.9938	R2 0.0241	0.9759	R2 0.2522	0.7478
Logl	392.9791		386.1737		296.1348	
AIC	-765.9583		-752.3474		-572.2696	
BIC	-732.826		-719.2154		-539.2864	

Table 10 presents the correlation coefficients between index returns and volume. We find that a negative relationship exists between stock index returns and trading volume in developed stock markets (FTSE100 and S&P500 indices) while the relationship is positive for emerging stock market(NSE20 index).

Table 10: Correlation between indices return and volume

	FTSE index	NSE20 index	S&P500 index
Daily	-0.050	0.008	-0.036
Weekly	-0.070	0.068	-0.121
Monthly	-0.198	0.138	-0.202

CONCLUSION

In this paper, the dynamic relationship between stock returns, volatility and trading volume in both developed and emerging stock markets is investigated. The results we find in this paper provide evidence that the behavior of the three stock indices returns and volume has common characteristics of many daily, weekly and monthly financial time series. The data exhibits a considerable kurtosis, which can be related to the time- dependence in conditional variance and also the distribution of all the time series (returns and volume) is relatively asymmetric. As a result of these two characteristics, all the time series data for returns and volume, shows a significant departure from normality and existence of conditional heteroscedasticity. All the series exhibits asymmetric behavior in the conditional variance which is related to leverage effects by many authors. The study finds strong evidence of volatility clustering, leverage effects and leptokurtic distribution for the indices returns and volume. The regime switching and MSGARCH models reveal that the two series oscillate between two regimes, that is, regime one(low volatility) and regime two (high volatility). Regime one is characterized by low unconditional volatility and low persistence of volatility while regime two is characterized by high unconditional volatility and high persistence of the volatility process. Moreover, we find negative relationship between stock returns and volume in developed stock markets (FTSE100 and S&P500 indices) and a positive relationship in emerging stock market. Finally, we observe that MSGARCH(1,1) is a better model compared to GARCH(1,1) as revealed by the log-likelihood, Akaike Information Criterion(AIC) and Bayesian Information Criterion(BIC) values.

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