# MACHAKOS UNIVERSITY 

University Examinations 2020/2021 Academic Year
SCHOOL OF PURE AND APPLIED SCIENCES

## DEPARTMENT OF MATHEMATICS AND STATISTICS FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR <br> BACHELOR OF SCIENCE (MATHEMATICS) <br> KCU 101: FUNDAMENTALS OF MATHEMATICS

DATE: 25/3/2021
TIME: 8.30-10.30 AM
INSTRUCTIONS

## Answer question ONE and any other TWO questions

## QUESTION ONE (30 MARKS)

a) Using remainder theorem solve the equation $2 x^{2}-x^{2}-7 x+6=0 \quad$ (3 marks)
b) Resolve the partial fraction $\frac{4(x-4)}{x^{2}-2 x-3}$
c) Evaluate $24+e^{2 x}=45$
d) Find the inverse of the $\mathrm{f}: x \rightarrow \sqrt{\frac{1+x}{1-x}} ;-1<x<1$
e) Express $-3<1-2 x \leq 3$
f) Using Cramer's Rule solve the following system of linear equations,

$$
\begin{aligned}
& 2 x_{1}+5 x_{2}+3 x_{3}=130 \\
& 4 x_{1}-7 x_{2}+x_{3}=-34 \\
& 5 x_{1}-2 x_{2}+3 x_{3}=74
\end{aligned}
$$

g) $\quad$ Minimize $-x_{1}-x_{2}$

Subject to $x_{1}+2 x_{2} \leq 3$

$$
\begin{aligned}
& 2 x_{1}+x_{2} \leq 3 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

h) Evaluate ;
i. $\quad \frac{d}{d x}(4 \ln 9 x)$
(2 marks)
ii. $\quad \int_{0}^{2}(2 x+3)^{7} d x$
i) The probability of a plant failing due to excess temperature 0.05 , due to excessive humidity is 0.02 and due to excess rainfall is 0.04 . Determine the probability that the plants will fail anyway.
(2 marks)

## QUESTION TWO (20 MARKS)

a) Using the power series of $e^{x}$ evaluate $x^{\frac{1}{2}}\left(2 e^{x^{2}}\right)$
b) Resolve the partial fraction

$$
\frac{3\left(2 x^{2}-8 x-1\right)}{(x+4)(x+1)(2 x-1)}
$$

c) Given the matrix $\boldsymbol{A}=\left[\begin{array}{ccc}2 & 1 & 3 \\ 3 & -2 & 5 \\ 4 & 3 & 2\end{array}\right]$

Using the cofactor method find $A^{-1}$, the inverse of A and hence determine the solution of the system,
(8 marks)

$$
\begin{gathered}
2 x_{1}+x_{2}+3 x_{3}=54 \\
3 x_{1}-2 x_{2}+5 x_{3}=52 \\
4 x_{1}+3 x_{2}-2 x_{3}=22
\end{gathered}
$$

## QUESTION THREE (20 MARKS)

a) A green farm uses at least 800 kg of special food every day. The special feed is a mixture of corn and soybeans meal with the following composition.

|  | A kg per kg of the feed stuff |  |  |
| :--- | :--- | :--- | :--- |
| Food stuff | Protein | Fiber | Cost ( Shs. per kg) |
| Corn | 0.09 | 0.02. | 30 |
| Soybeans | 0.06 | 0.06 | 90 |

The dietary requirements of the total feed stipulate at least $30 \%$ protein and at most $5 \%$ fiber. Determine the daily minimum cost feed mix.
b) Determine the maximum area of the rectangular farm that can be enclosed by wire mesh 1200 long.

## QUESTION FOUR (20 MARKS)

a) Evaluate the following integrals
i. $\int \frac{x^{2}+7 x+10}{x+5} d x$ (5 marks)
ii. $\quad \int e^{3 x-2} d x$
b) Differentiate the following
i. $x^{2} e^{2 x}$
ii. $(2 x-1)^{2}$

## QUESTION FIVE (20 MARKS)

a) The demand for farm tools in a given store is, on average, five times a day and the demand follows the Poisson distribution. How many of these tools should be kept in the store so that the probability of there being an available tool is greater than $10 \%$.
b) The following shows the distribution of body weights of calves at the first lactations.

| Body weight (kg) | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No. of calves | 12 | 26 | 34 | 20 | 8 |

Determine;

| i. | Arithmetic Mean | (3 marks) |
| :--- | :--- | :--- |
| ii. | Geometric Mean | $(3$ marks $)$ |
| iii. | Harmonic Mean | $(3$ marks $)$ |
| iv. | Standard deviation | $(4$ marks $)$ |
| v. | Coefficient of variation | $(2$ marks $)$ |

