MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR THIRD SEMESTER EXAMINATION FOR

DIPLOMA IN MECHANICAL ENGINEERING.

MATHEMATICS 1

DATE: 15/12/2020 **TIME: 11.30-2.30 PM**

INSTRUCTIONS

The paper consists of EIGHT questions. Answer any FIVE questions.

ALL questions carry equal marks.

Show all your working

1. a) evaluate the middle term in the binomial expansion of $(2x + 3y)^8$ and determine its value when $x = \frac{1}{3}$ and $y = \frac{1}{2}$. (7 marks)

   b) Find the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{\frac{-1}{3}}$, and state the values of x for which the expansion is valid. (3 marks)

   c) (i) Use the binomial theorem to expand $\sqrt{(\frac{1-x}{1+x})}$ as far as the fourth term.

   (ii) By setting $x = \frac{1}{15}$ in first three terms in (i), determine the approximate value of $\sqrt{14}$ corect to four decimal places. (10 marks)

2. (a) Simplify the expression: $\frac{(1-x)^{\frac{1}{2}} - x(1-x)^{-\frac{1}{2}}}{1-x}$ (3 marks)

   (b) Evaluate: $2^{(2x+2)} - 7(2^x) + 3 = 0$, correctto four decimal places. (10 marks)

   (c) Three forces, F_1, F_2 and F_3 in newtons, necessary for the equilibrium of a mechanical system satisfy the simultaneous equations:

$F1 - 2F2 + F3 = 1$

$-2F1 + 3F2 + 2F3 = 8$

$3F1 + 4F2 - 3F3 = 5$

Use the method of elimination to solve the equations. (7 marks)

3. (a) Given that $5\cosh x + 3\sinh x = pe^{x} + qe^{-x}$, determine the value of p and q. (4 marks)

(b) Prove the identities:

(i) $\text{cosech}^2 x = \coth^2 x - 1$

(ii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$ (8 marks)

(c) Solve the equation:

$2\cosh x - 3\sinh x + 1 = 0$, correct to four decimal places. (8 marks)

4. (a) Simplify: $(x^2 -)^2 \times \sqrt{(x+1)} \div (x-1)^{\frac{-3}{2}}$. (3 marks)

(b) Solve the equations:

(i) $\log(x-3) + \log(x+3) = 2\log(x+3)$

(ii) $4(16^{x+4}) \times 5.2^{2x} = 13$ (9 marks)

(c) Solve the equation $\log_5 y + \log_y 25 = 3$. (8 marks)

5. (a) The sum of the first four terms of an arithmetic progression is 4, and the difference between the eighth and fourth terms is 12. Determine the:

(i) First term and common difference;

(ii) Sum of the first twenty one terms. (7 marks)

(b) The ratio of the fourth and sixth terms of a geometric progression is 4, and the sum of the first three terms is $\frac{21}{4}$. Determine the:

(i) First term and common ratio

(ii) Sum to infinity of the geometric progression. (7 marks)

(c) Determine the polar equation of the parabola $x^2 = 4(1 + y)$. (6 marks)

6. (a) Determine the inverse of $f(x) = \frac{x+4}{2x-5}$. (4 marks)

(b) Convert:

(i) $r = 3(1 + 2\cos\theta)$ to cartesian form

(ii) $x^2 + y^2 = 7x$ to polar form (7 marks)

(c) Solve the equation $\log_3 x + \log_x 9 = 3$ (9 marks)

7. (a) Express $\frac{2+3j}{3+4j}$ in the form a + bj. (3 marks)

(b) If Z = x + yj, find the values of x and y such that

$\frac{1}{z} + \frac{2}{z} = 1 + j$ (6 marks)

(c) Use the De Moivre's theorem to show that

$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ (5 marks)

(d) Simplify $3 + 2j + 5(3 - j) + j(3j - 4)$ expressing the result in the polar form. (6 marks)

8. (a) Solve that the following equations for $0^0 \ll x \ll 360^0$ given

(i) Tan 2x = 1

(ii) $\text{Cos}^2 x = \frac{1}{4}$

(iii) 3cosx + 2 = 0 (10 marks)

(b) show that $\frac{1}{1+sin\theta} + \frac{1}{1-sin\theta} = 2sec^2\theta$ (5 marks)

(c) Given that $\text{Sin}A = \frac{12}{13}$ and $\text{Cos } B = \frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) Sin (A – B)

ii) Tan (A + B) (5 marks)