MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR THIRD SEMESTER EXAMINATION FOR

DIPLOMA IN ELECTRICAL ENGINEERING

MATHEMATICS 1

DATE: 15/12/2020 **TIME: 11.30-2.30 PM**

INSTRUCTIONS

The paper consists of EIGHT questions. Answer any FIVE questions.

ALL questions carry equal marks.

Show all your working

1. (a) Solve the following equation for all values of θ between 0^0 and 360^0.

   $2\sin\theta - 3\cos\theta = 2$ (7 marks)

   (b) Solve for x in the following equations

   i. $\log_3(2x - 3) = -1$; (3 marks)

   ii. $3^{2x} = 4(3^x) + 3$. (4 marks)

   (c) Find the minimum and maximum ordinates of the curve $y = x^2(x + 1)$ and classify them. (6 marks)

2. (a) Given that $(a + b) + j(a - b) = 9 + j10$ (4 marks)

   (b) Find the equation of the tangent to the curve y = x2 – x – 2 at the point (1,-2). (5 marks)

   (c) A rectangular sheet of metal having dimensions 20cm by 12cm has squares removed From each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box. (11 marks)

3. (a) Find the derivative of the function $y = 2x^2$ by first principals. (5 marks)

   (b) (i) Given $Z = \frac{1}{2+j3} + \frac{1}{1-j2}$, express Z in the form a + jb. (5 marks)

(ii) Express Z = (2 – j7) in polar form. (4 marks)

(c) Determine the term in x^5 in the binomial expansion of $(2x + 3y)^9$, and find the value when $x = \frac{1}{3}$ and $y = \frac{1}{2}$. (6 marks)

4. (a) Given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) Sin (A – B)

ii) Tan (A + B)

(b) Prove the identities:

i. $\frac{1-\cos\theta}{\sin\theta} + \frac{\sin\theta}{1-\cos\theta} = 2\operatorname{Cosec}\theta$

ii. $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$ (8 marks)

(c) Given $t = \tan 22\frac{1}{2}^0$

i. Show that $\tan 45^0 = \frac{2t}{1-t^2}$;

ii. Hence solve the equation:

$t^2 + 2t - 1 = 0$, leaving your answer in surd form. (7 marks)

5. (a) Determine how many ways in committees can be formed from a set of 5 governors and 7 senators if each committee consists of 3 governors and 4 senators. (5 marks)

(b) Solve the equation; $2\log_9(x + 1) + \log_3 x = 1$ (7 marks)

(c) Three forces F1, F2 and F3 in newtons, necessary for the equilibrium of a certain mechanical system satisfy the simultaneous equations:

$F_1 - 2F_2 + F_3 = 1$

$F_1 + 3F_2 - 2F_3 = 2$

$F_1 + F_2 + F_3 = 7$

Use elimination method to solve the equation. (8 marks)

6. (a) Evaluate the integrals

(i) $\int_1^2 \frac{\ln^2 x}{x} dx$

(ii) $\int_0^1 \frac{x dx}{(1+x)(1+x^2)} dx$ (13 marks)

(b) Use integration to determine the area of the region in the first quadrant bounded by the curve $y = 2x^2$ and the line $y = 4x$. (7 marks)

7. (a) Given that pCoshx + qSinhx = $3e^x - 2e^{-x}$, determine the values of p and q. (7 marks)

(b) Prove the identities

(i) $\text{Cosh}2x = \frac{1+tanh^2x}{1-tanh^2x}$

(ii) $\tanh 3x = \frac{3\,tanhx+tanh^3x}{1+3\,tanh^2x}$ (6 marks)

(c) Solve the equation :

3coshx – 7sinhx = 2, correct to three decimal places. (7 marks)

8. (a) If $Z = 5x^4 + 2x^3y^2 - 3y$, determine:

(i) $\frac{бz}{бx}$; (ii) $\frac{бz}{бy}$. (6 marks)

(b) Use implicit differentiation to determine the equation of the normal to the curve

$X^2 + y^2 - 4xy + 6x + 4y = 8$, at the point (1,1). (7 marks)

(c) Determine the stationary points of the function $f(x) = 2x^3 + 3x^2 - 12x + 6$ and state their nature. (7 marks)