MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

CSO 108: DISCRETE MATHEMATICS

DATE: **TIME:**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY THREE Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Define the following terms.

i. A graph

ii. trail

iii. Set

iv. Proposition

v. A subset (5 marks)

b) Find the power set of the set $A = \{1,2,3,4\}$. (3 marks)

c) Construct all the unlabeled graphs with 4 vertices (5 marks)

d) Construct the truth table for the disjunction of two proposition (4 marks)

e) In how many distinguishable ways can the product $Z^7X^8Y^7T^6$ be arranged without using exponents. (3 marks)

f) Given $A = ((abc)'c)'(a' + c)(b' + ac')'$ express it as a sum of product expression. (5 marks)

g) Prove that if a bipartite graph has a cycle then all its cycles are of even length. (5 marks)

QUESTION TWO (20 MARKS)

a) Construct the logic circuit for the following output $Y = (XY + ZY')' + (X' + ZY)'$ (5 marks)

b) Show that D_{210} (where D_{210} are divisors of 210) is a Boolean algebra

i) Find the atoms (3 marks)

ii) Find the subalgebra (3 marks)

iii) Construct the lattice diagram (4 marks)

c) Given that $= 00111001$ $Y = 11100011$ $Z = 00110010$ $T = 01011011$. Find

$A = X.Y.Z.T + TZ$ (5 marks)

QUESTION THREE (20 MARKS)

a) Given that a and b are rational with b$\neq$ 0 and s is an irrational number such that $a - bs = t$.

Show that t is irrational hence show that $\frac{-1+\sqrt{3}}{1+\sqrt{3}}$ is irrational (6 marks)

b) Show that $\sqrt{5}$ is irrational (5 marks)

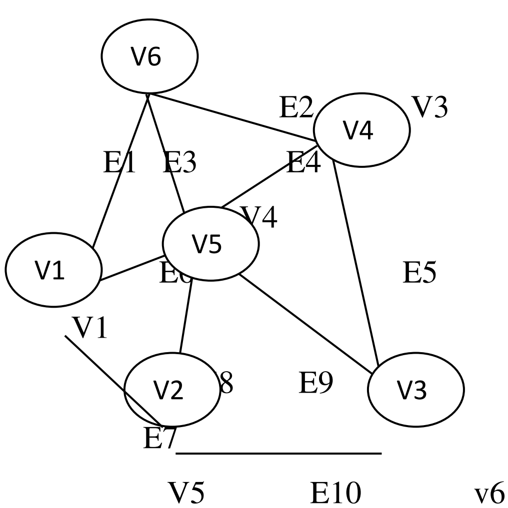
c) Proof that set of all even natural numbers is countable. (5 marks)

d) Suppose two boys say Fred and Sum are playing a Football tournament such that the first person to win two games in a row or who wins a total of three games wins the tournament. Construct a rooted tree to illustrate the above. Find the number of ways the tournament can be won. (5 marks)

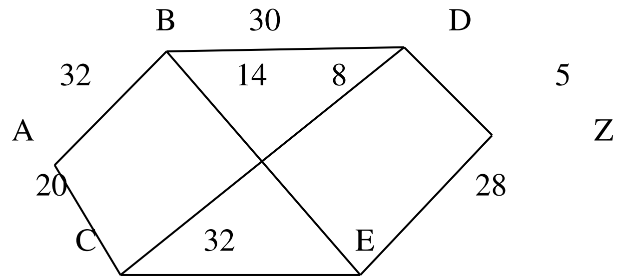
QUESTION FOUR (20 MARKS)

a) Prove that if G is a connected planar graph with P vertices and q edges. Where $p \geq 3$. then $q \leq 3p - 6$. (5 marks)

b) Construct the Incidency matrix for the following graph. (5 marks)



c) Find the shortest path from A to Z for the map shown below (5 marks)



d) Prove that if M is a map with V vertices, E edges and R regions and K components. Then $V - E + R = K + 1$. (5 marks)

QUESTION FIVE (20 MARKS)

a) Let $U = \{i, j, k, l, m, n, o, p, q, r, s, t, u\}$, $A = \{i, k, l, m, q\}$ $B = \{j, k, q, r\}$ $C = \{j, k, m, o\}$ and $D = \{j, o, p\}$.

Determine the set

i) $A \cup B$

ii) $A \cap C$

iii) $(A \cup B) \cap C^c$

iv) $(C \cap A) \cup D$

v) (8 marks)

b) Let $A = \{s, t\}$ and $B = \{1,4,6\}$ determine the set $(A \times B)\ X\ B$ (2 marks)

c) A man who works five days a week can travel to work on foot, by bicycle or by bus. In how many ways can he arrange a week's travelling to work? (6 marks)

d) Show that $\rceil p \vee q$ and $p \to q$ are logically equivalent (6 marks)