MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 101 FUNDAMENTALS OF ACTUARIAL MATHEMATICS

DATE: 25/2/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:
Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) You are given a discount function v where $v(1,3) = 0.9, v(3,6) = 0.8, v(8,6) = 1.2$

i. How much must you invest at time 1, in order to accumulate 10 at time 8? (3 marks)

ii. If you invest 100 at time 3, how much will have accumulated by time 8? (3 marks)

b) You are given that $q_{60} = 0.20, q_{61} = 0.25, q_{62} = 0.25, q_{63} = 0.30, q_{64} = 0.40$.

i. Find ℓ_x for ages $60 - 65$, beginning with $\ell_{60} = 1000$. (3 marks)

ii. Find the probabilities of the following:

I. (61) will die between the ages of 62 and 64. (3 marks)

II. (62) will live to age 65. (3 marks)

c) Given that $e_{65} = 0.8, find\ e_x\ for\ x = 60 - 64$ (4 marks)

d) A group of individuals age 40, each invest 1000 in a fund earning interest at 5%. At the end of 20 years the fund is divided equally among the survivors. If $20P_{40}$ =0.8, how much does each get? (4 marks)

e) Find the price of $a\ Kshs.1{,}000$ par value $10 - year$ bond with coupons at 8.4% payable semi-annually , which will be redeemed at $Kshs.1050$. The bond is bought to yield 10% convertible semi-annually. Use the basic formula. (4 marks)

f) Calculate the time in days for £1,500 to accumulate to £1,550 at

i. Simple rate of interest of 5% per annum (3 marks)

ii. A force of interest of 5% $p$er annum. (3 marks)

QUESTION TWO (20 MARKS)

a) A loan of £$50,000$ is to be repaid over 5 years by a level annuity payable annually in arrears. The amount of the annual payment is calculated on the basis of an interest rate of 3% per annum effective. Immediately after the third repayment was made, the borrower requests that he be able to pay off the loan with a single lump sum. Use a repayment schedule to calculate the value of the lump sum required to repay the loan at this time. (6 marks)

b) Differentiate between the following ways of calculating the outstanding loan:

i. Prospective approach. (2 marks)

ii. Retrospective approach (2 marks)

c) Person A borrows $10,000 from Person B and agrees to repay it with equal quarterly instalments of principal and interest at 8% convertible quarterly over six years. At the end of two years Person B sells the right to receive future payments to Person C at 10% convertible quarterly. Find the total amount of interest received:

i. By Person C. (5 marks)

ii. By Person B. (5 marks)

QUESTION THREE (20 MARKS)

a) An investor buys an $n - year$ annuity with a present value of $Kshs. 1{,}000$ computed at 8%. The investor pays a price which will permit the replacement of the original investment in a sinking fund earning 7% and will also produce an overall yield rate of 9% on the entire transaction. Find the price which the investor should pay for the annuity. (10 marks)

b) A loan of £10,000 is to be repaid over 10 years by a level annuity payable monthly in arrears. The amount of the monthly payment is calculated on the basis of an interest rate of 1% per month effective. Find:

i. The monthly repayment, (4 marks)

ii. The total capital repaid and interest paid in:

I. The first year and (3 marks)

II. The final year, (3 marks)

QUESTION FOUR (20 MARKS)

a) An asset is being depreciated over a 10-year period. It has no salvage value at the end of the ten years, that is S=0. If the depreciation charge in the third year is Kshs. 1000, find the depreciation charge in the ninth year:

   i. By the sinking fund method, assuming j=0.05. (3 marks)

   ii. By the straight-line method. (2 marks)

   iii. By the sum of the years digit method. (3 marks)

b) Why can the declining balance method not be used? (1 mark)

c) Find the original value of the asset in each of the above cases in (a) (7 marks)

d) Define the terms:

   i. Life annuity (1 mark)

   ii. Life insurance (1 mark)

   iii. Life tables (1 mark)

   iv. The theory of Consistency (1 mark)

   v. The probability of surviving (1 mark)

QUESTION FIVE (20 MARKS)

a) A loan is repaid with repayments which start at $200 the first year and increase by $50 per year until a payment of $1,000 is made, at which time payments cease. If interest is 4% effective, find the amount of principal in the fourth payment. (6 marks)

b) An investor in common stock measures investment returns annually using an effective rate of interest. The investor earns 15% during the first year ,-5% during the second year and 8% during the third. Find the equivalent level effective rate of return over the three-year period. (4 marks)

c) Find the present value of $5,000 to be paid at the end of 25 months at a rate of discount of 8% convertible quarterly:

   i. Assuming compound discount throughout. (3 marks)

   ii. Assuming simple discount during the final fractional period. (4 marks)

d) Find the level effective rate of interest over a three-year period which is equivalent to an effective rate of discount of 8% the first year, 7% the second year, and 6% the third year. (3 marks)