

MACHAKOS UNIVERSITY

University Examinations for 2019/2020 Academic Year SCHOOL OF PURE AND APPLIED SCIENCES DEPARTMENT OF MATHEMATICS AND STATISTICS FIRST YEAR THIRD SEMESTER EXAMINATION FOR DIPLOMA IN MECHANICAL ENGINEERING.

MATHEMATICS 1

DATE: 15/12/2020 TIME: 11.30-2.30 PM

INSTRUCTIONS

The paper consists of EIGHT questions. Answer any FIVE questions.

ALL questions carry equal marks.

Show all your working

- 1. a) evaluate the middle term in the binomial expansion of $(2x + 3y)^8$ and determine its value when $x = \frac{1}{3}$ and $y = \frac{1}{2}$. (7 marks)
 - b) Find the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{\frac{-1}{3}}$, and state the values of x for which the expansion is valid. (3 marks)
 - c) (i) Use the binomial theorem to expand $\sqrt{(\frac{1-x}{1+x})}$ as far as the fourth term.
 - (ii) By setting $x = \frac{1}{15}$ in first three terms in (i), determine the approximate value of $\sqrt{14}$ corect to four decimal places. (10 marks)
- 2. (a) Simplify the expression: $\frac{(1-x)^{\frac{1}{2}} x(1-x)^{-\frac{1}{2}}}{1-x}$ (3 marks)
 - (b) Evaluate: $2^{(2x+2)} 7(2^x) + 3 = 0$, correctto four decimal places. (10 marks)
 - (c) Three forces, F_1 , F_2 and F_3 in newtons, necessary for the equilibrium of a mechanical system satisfy the simultaneous equations:

F1 - 2F2 + F3 = 1

$$-2F1 + 3F2 + 2F3 = 8$$

$$3F1 + 4F2 - 3F3 = 5$$

Use the method of elimination to solve the equations.

(7 marks)

- 3. (a) Given that $5\cosh x + 3\sinh x = pe^x + qe^{-x}$, determine the value of p and q. (4 marks)
 - (b) Prove the identities:
 - (i) $\operatorname{cosech}^2 x = \coth^2 x 1$

(ii)
$$\cosh 3x = 4\cosh^3 x - 3\cosh x$$
 (8 marks)

(c) Solve the equation:

$$2\cosh x - 3\sinh x + 1 = 0$$
, correct to four decimal places. (8 marks)

- 4. (a) Simplify: $(x^2)^2 \times \sqrt{(x+1)} \div (x-1)^{\frac{-3}{2}}$. (3 marks)
 - (b) Solve the equations:
 - (i) $\log(x-3) + \log(x+3) = 2\log(x+3)$

(ii)
$$4(16^{x+4}) \times 5.2^{2x} = 13$$
 (9 marks)

- (c) Solve the equation $\log_5 y + \log_y 25 = 3$. (8 marks)
- 5. (a) The sum of the first four terms of an arithmetic progression is 4, and the difference between the eighth and fourth terms is 12. Determine the:
 - (i) First term and common difference;
 - (ii) Sum of the first twenty one terms. (7 marks)
 - (b) The ratio of the fourth and sixth terms of a geometric progression is 4, and the sum of the first three terms is $\frac{21}{4}$. Determine the:
 - (i) First term and common ratio
 - (ii) Sum to infinity of the geometric progression. (7 marks)
 - (c) Determine the polar equation of the parabola $x^2 = 4(1 + y)$. (6 marks)
- 6. (a) Determine the inverse of $f(x) = \frac{x+4}{2x-5}$. (4 marks)
 - (b) Convert:
 - (i) $r = 3(1 + 2\cos\theta)$ to cartesian form

(ii)
$$x^2 + y^2 = 7x$$
 to polar form (7 marks)

- (c) Solve the equation $log_3x + log_x9 = 3$ (9 marks)
- 7. (a) Express $\frac{2+3j}{3+4j}$ in the form a + bj. (3 marks)

(b) If Z = x + yj, find the values of x and y such that

$$\frac{1}{z} + \frac{2}{z} = 1 + j$$
 (6 marks)

(c) Use the De Moivre's theorem to show that

$$\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1 \tag{5 marks}$$

- (d) Simplify 3 + 2j + 5(3 j) + j(3j 4) expressing the result in the polar form.(6 marks)
- 8. (a) Solve that the following equations for $0^0 \ll x \ll 360^0$ given
 - (i) $\operatorname{Tan} 2x = 1$
 - (ii) $Cos^2x = \frac{1}{4}$
 - (iii) $3\cos x + 2 = 0$ (10 marks)
 - (b) show that $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$ (5 marks)
 - (c) Given that SinA = $\frac{12}{13}$ and Cos B = $\frac{4}{5}$ where A is obtuse and B is acute, determine the values of;
 - i) Sin(A B)
 - ii) $\operatorname{Tan}(A + B)$ (5 marks)