MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SST 103: LINEAR ALGEBRA

DATE: 29/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

Answer Question ONE and any other TWO Questions.

QUESTION ONE 30 Marks (Compulsory)

a) Find the angle between the following vectors $4i + 4j + 6k$ and $i + 2j + 2k$ (4 marks)

b) Solve the homogenous system by Gauss elimination method

$$2x - 4y + 6z = 20$$
$$3x - 6y + z = 22$$
$$-2x + 5y - 9z = 18$$

(5 marks)

c) If $\begin{vmatrix} 4y & 20 \\ 4y & 8y+4 \end{vmatrix} = 8$ find y (3 marks)

Calculate the cross product of the vectors $\vec{u} = (2, 2, 3)$ and $\vec{v} = (-1, 1, 3)$. (3 marks)

d) Find the inverse of the following matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

(4 marks)

e) Show that the vector (2,-6,3) is a linear combination of the (3,-5,4) and (1,-2,-1)

f) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, −1). (4 marks)

g) Determine if (1,2,2,1), (2,3,4,1) and (3,8,7,5) is linearly dependent (4 marks)

h) Find the dot product of the following vectors $(3,4,-1)$ and $(4,6,1)$ (3 marks)

QUESTION TWO (20 MARKS)

a) Let $T: R^2 \rightarrow R$ be defined by $T(x,y) = x + y$. Is T a linear map? (7 marks)

b) Let $T: R^2 \rightarrow R^2$ be defined by $T(x,y) = \{x + y, x\}$. Find the kernel of T (5 marks)

c) Let $T: R^3 \rightarrow R^2$ be the linear mapping defined by

$T(x,y,z) = (x + 2y - z, y + z, x + y + z)$. Find the basis and dimension of

i. Image of T

ii. Kernel of T (8 marks)

QUESTION THREE (20 MARKS)

a) Find the equation of the plane passing through the point $(3,-1,7)$ and perpendicular to the vector $n = (4,2,-5)$ (5 marks)

b) Find the distance from the origin to the plane $2x + 3y - z = 2$ (5 marks)

c) Find the parametric equations for the line of intersection of the plane

$3x + 2y - 4z - 6 = 0$ and $x - 3y - 2z - 4 = 0$ (5 marks)

d) Find the distance D between the point $(1,-4,-3)$ and the plane $2x - 3y + 6z = -1$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Show that $w = \{ (x,y,z) | x + y + z = 0)\}$ is a subspace of $\mathbb{R}^3$ (5 marks)

b) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent. (5 marks)

c) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$ (5 marks)

d) Solve by crammer's Rule

$$x - 2y + 3z = 10$$
$$3x - 6y + z = 22$$
$$-2x + 5y - 2z = -18$$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Define a vector space (6 marks)

b) Show that W= $\{(x, y) \,/\, x = 2y\}$ is a subspace for R^2. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)