MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR BACHELOR OF ………………………………………………

ECU 300: ENGINEERING MATHEMATICS IX

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Answer question **ONE** (**compulsory**) and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) Find the first and the second partial derivatives of the function $f(x, y) = 2x^3y^2 + y^3$ (5 marks)

b) Find the total differential of the function $f(x, y) = ye^{x+y}$ (4 marks)

c) Show that $(y + z)dx + xdy + xdz$ is an exact differential (4 marks)

d) Given that $x(u) = 1 + au$ and $y(u) = bu^3$. Find the rate of change of $f(x, y) = xe^{-y}$ with respect to u (5 marks)

e) Show that the differential $df = x^2dy - (y^2 + xy)dx$ is not exact but that $dg = (xy^2)^{-1}df$ is exact (6 marks)

f) Evaluate the double integral $I = \iint_R x^2ydxdy$ where R is the triangular area bounded by the lines $x = 0, y = 0$ and $x + y = 1$. Reverse the order of integration and demonstrate that the same result is obtained (6 marks)

QUESTION TWO (20 MARKS)

a) The moment of inertia of a right circular cone of height, h and radius, a about the $z - axis$ is given by

$$I = \int_{\theta=0}^{2\pi} \int_{r=0}^{a} \int_{z=0}^{\frac{h(a-r)}{a}} \sigma r^3 dr d\theta dz$$

And the total mass of the cone is

$$M = \int_{\theta=0}^{2\pi} \int_{r=0}^{a} \int_{z=0}^{\frac{h(a-r)}{a}} \sigma r dr d\theta dz$$

Show that $I = \frac{3}{10} M a^2$ (12 marks)

b) Find the angle between the vectors $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ (4 marks)

c) Find the area A of the parallelogram with sides $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$ (4 marks)

QUESTION THREE (20 MARKS)

a) The position vector of a particle at time t in Cartesian coordinates is given by $\boldsymbol{r}(t) = 2t^2\boldsymbol{i} + (3t-2)\boldsymbol{j} + (3t^2-1)\boldsymbol{k}$. find the velocity of the particle at $t = 1$ and the components of its acceleration in the direction

$$\boldsymbol{s} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$$

(8 marks)

b) The position vector of a particle in plane polar coordinates is $\boldsymbol{r}(t) = \rho(t)\hat{\boldsymbol{e}}_\rho$ find the expressions for the velocity and acceleration of the particle in this coordinates (8 marks)

c) Find the gradient of the scalar field $\phi = xy^2z^3$ at the point $(1,1,1)$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Find the divergence of the vector field $\boldsymbol{a} = x^2y^2\boldsymbol{i} + y^2z^2\boldsymbol{j} + x^2z^2\boldsymbol{k}$ at the point $(2,1,3)$ (4 marks)

b) Find the Laplacian of the scalar field $\phi = xy^2z^3$ at the point $(1,4,2)$ (4 marks)

c) Find the curl of the vector field $\boldsymbol{b} = x^2y^2z^2\boldsymbol{i} + y^2z^2\boldsymbol{j} + x^2z^2\boldsymbol{k}$ (5 marks)

d) Express the vector field $\boldsymbol{p} = yz\boldsymbol{i} - y\boldsymbol{j} + xz^2\boldsymbol{k}$ in cylindrical polar coordinates and hence calculate its divergence (7 marks)

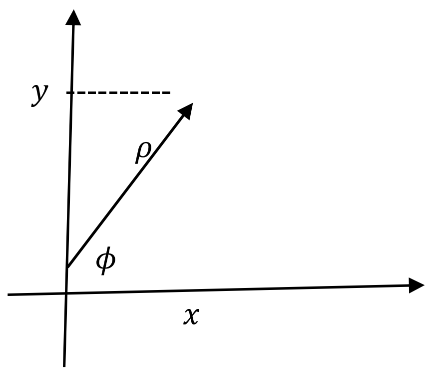
QUESTION FIVE (20 MARKS)

a) Using change of variable techniques, evaluate the integral

$$I = \frac{1}{2}\pi\sigma \int_{z=0}^{h} \left\{a\left(\frac{h-z}{h}\right)\right\}^4 dz$$ (10 marks)

b) Plane polar coordinates, ρ and ϕ and Cartesian coordinates x and y are related by the expressions $x = \rho\cos\theta$, $y = \rho\sin\theta$ as can be seen in the figure 1 below. An arbitrary function $f(x, y)$ can be re-expressed as a function $g(\rho, \phi)$. Transform the expression $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ in to one in ρ and ϕ

Figure 1



(10 marks)