MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 430: NUMERICAL ANALYSIS II

DATE: 20/1/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Solve the system of equations below using the inverse of the coefficients matrix method

$$x_1 + 2x_2 - x_3 = 2$$
$$3x_1 + 6x_2 + x_3 = 1$$
$$3x_1 + 3x_2 + 2x_3 = 3$$

(5 marks)

b) Determine the Eigen values and the corresponding Eigen vectors of the following system.

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

(5 marks)

c) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$,evaluate $f'(x)$ (5 marks)

d) Consider the initial value problem (I .v. p) given by

$y' = 2x + 3y$; $y(0) = 1$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$ (5 marks)

e) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i. Show that the matrix satisfies its own characteristics equation (5 marks)

ii. Determine A^{-1} (5 marks)

QUESTION TWO (20 MARKS)

Consider the differential equation $\frac{dy}{dx} = 2e^x y$, $y(0) = 2$. Calculate $y(0.2)$ using Adams predictor corrector formula by calculating $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ using the Euler's modified formula.

QUESTION THREE (20 MARKS)

Consider the approximate value of $I = \int_{-1}^{1} e^{-x^2} \cos x dx$ obtain the value using.

a) Gauss-Legendre integration method for $n = 2, 3$ (10 marks)

b) Radau integration method for $n = 2, 3$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Calculate the largest Eigen –value for the matrix

$$\begin{bmatrix} 10 & 4 & -1 \\ 4 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

Also evaluate the Eigen vector corresponding to the Largest Eigen -value (10 marks)

b) Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Where a is a real constant

i. Determine the values of a, for which the Jacobi and Gauss-Seidel methods converge. (7 marks)

ii. For $a = 0.5$ calculate the value of ω which minimizes the spectral radius of the SOR iteration method. (3 marks)

QUESTION FIVE (20 MARKS)

a) Solve the system of equations below using Doolittle's method.

(8 marks)

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

b) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

Using the Gauss- Elimination method with partial pivoting (7 marks)

c) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x} dx$ using Romberg integration with step size $h = \frac{1}{16}$ (5 marks)