MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF ……………………………..

SMA 432: PARTIAL DIFFERENTIAL EQUATION I

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory)*** and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Solve the differential equation $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$ (6 marks)

b) State the order of the partial differential equations below

i.) $\frac{\partial^2 z}{\partial x^2} = 1 + \left(\frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$ (3 marks)

ii.) $\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial^2 z}{\partial y^2}\right) = 2x\left(\frac{\partial z}{\partial x}\right)$ (3 marks)

c) State the degree of the following differential equations

i.) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{\frac{3}{4}}$ (4 marks)

ii.) $y\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial z}{\partial y}\right)^2 = 1 + \left(\frac{\partial z}{\partial y}\right)^{\frac{1}{2}}$ (2 marks)

d) Eliminate the arbitrary function f from the equation $f\left(x^2+y^2+z^2, z^2-2xy\right)=0$ (6 marks)

e) Solve the equation $\frac{xdx}{y^2z}=\frac{dy}{xz}=\frac{dz}{y^2}$ (6 marks)

QUESTION TWO (20 MARKS)

a) Using Jacobi's method determine a complete integral of $p_1x_1+p_2x_2=p_3^2$ (10 marks)

b) Consider the partial differential equation $p^2+q^2-2pq\tanh 2y=\sec h^2 2y$;

i.) Write its Charpit's auxiliary equations (2 marks)

ii.) Solve the p.d.e by considering its Charpit's auxiliary equations. (8 marks)

QUESTION THREE (20 MARKS)

Solve;

a) $\frac{dx}{x^2}=\frac{dy}{y^2}=\frac{dz}{nxy}$ (7 marks)

b) $\frac{dx}{y^2(x-y)}=\frac{dy}{-x^2(x-y)}=\frac{dz}{z(x^2-y^2)}$ (8 marks)

c) $\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{(x^2+y^2)}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Determine the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersection with the family of planes parallel to $z=c$ (8 marks)

b) Solve the semi-linear equation $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$ (7 marks)

c) Consider the partial differential equation $z=px+qy+p+q-pq$ with a complete integral of the form $z=ax+by+p+a+b-ab$; where a and b are arbitrary constants. Determine a general solution by finding the envelope of those planes that pass through the origin. (5 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the integral surface of the quasi-linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2 - y^2 z)$ which contains the straight line $x + y = 0$, $z = 1$

(8 marks)

b) in the one-dimensional unsteady flow of compressible fluid, the velocity u *and* c where $c^2 = \frac{dp}{d\rho}$, p is pressure and ρ is density satisfying the following equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + 2c\frac{\partial c}{\partial x} = 0$$

$$2\frac{\partial c}{\partial t} + 2u\frac{\partial c}{\partial x} + c\frac{\partial u}{\partial x} = 0$$

Prove that the characteristics are given by the differential equation $dx = (u \pm c)dt$ and that on the characteristic $dx = (u + c)dt$, $u + 2c$ is a constant. (12 marks)