MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR BACHELOR OF ……………………………………..

SMA 435: METHODS OF FLUID MECHANICS

DATE: **TIME:**

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Given that $f(\varepsilon) = \sin e^{\varepsilon}$, obtain a three-term asymptotic expansion for $f(\varepsilon)$ as $\epsilon \to 0$ (4 marks)

b) Consider the functions $\phi_1(\varepsilon)$, $\phi_2(\varepsilon)$, ... Given that the functions are well ordered as $\epsilon \to 0$, state the necessary condition that $f(\varepsilon)$ is an asymptotic expansion. (2 marks)

c) Given that $\phi_1 = 1$, $\phi_2 = \varepsilon$ and $\phi_3 = \varepsilon^2 \ldots$, and $f(\varepsilon) = \frac{1}{1+\varepsilon} + e^{-1/\varepsilon}$ obtain a three-term asymptotic expansion for $f(\varepsilon)$. (5 marks)

d) Consider the function, $\Psi(r,\theta)$ where $r^2 = x^2 + y^2$, $x = r\cos\theta$ and $y = r\sin\theta$, show that; $\frac{\partial r}{\partial x} = \cos\theta$, $\frac{\partial r}{\partial y} = \sin\theta$, $\frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}$ and $\frac{\partial \theta}{\partial x} = -\frac{\sin\theta}{r}$ (8 marks)

e) Compute the Laplace transform of $f(t) = e^{at}$ (5 marks)

f) Using finite element methods, determine the approximate solution of the equation;

$-\frac{d}{dx}\left(x\frac{du}{dx}\right) + u = 0$ for $0 \le x \le 1$ subject to $u(0) = 1, \left(x\frac{du}{dx}\right)_{x=1} = 0$ (6 marks)

QUESTION TWO (20 MARKS)

a) Consider the equation $\varepsilon x^2 + 2x - 1 = 0$;

i. Define a singular perturbation. (1 mark)

ii. By sketching the suitable functions, state the number of real-valued solutions and approximate their locations. (4 marks)

iii. Obtain a two-term asymptotic expansion for the solution of x for a small ε.(5 marks)

b) Find the asymptotic approximation of the solution of the differential equation

$\frac{dy}{dx} = f(x,y)$, where $y = (u,q)^T$and $f = \begin{cases} 1 - ue^{\varepsilon(q-1)} \\ ue^{\varepsilon(q-1)} - q \end{cases}$ and $y(0) = 0$ (10 marks)

QUESTION THREE (20 MARKS)

a) The Friedrichs model problem for boundary layer in viscous fluid is given by $\varepsilon y'' = a - y'$ for $0 < x < 1$, where $y(0) = 0$ and $y(1) = 1$ and a is a positive constant with$a \neq 1$. Derive a composite expansion of the solution of the problem. (10 marks)

b) Obtain the composite expansion of the solution of the differential equation $\varepsilon y'' + 3y' + y^3 = 0$ for $0 < x < 1$, where $y(0) = 0$ and $y(1) = \frac{1}{2}$. (10 marks)

QUESTION FOUR (20 MARKS)

a) Solve the Laplace equation $\nabla^2 \Psi = 0$ on a disk of radius 3 units subject to the boundary conditions; $\Psi(3,\theta) = \begin{cases} 1, & 0 \leq \theta \leq \pi \\ \sin^2\theta, & \pi \leq \theta \leq 2\pi \end{cases}$ (8 marks)

b) Find the Laplace transform of $f(t) = \begin{cases} 1, 0 \leq t \leq 2 \\ t-2, \ \ t \geq 2 \end{cases}$ (4 marks)

c) Solve the initial value problem by Laplace transform.

$y'' + 9y = 6\cos 3t;\ y(0) = 0, y'(0) = 0$ (8 marks)

QUESTION FIVE (20 MARKS)

Consider the differential equation;

$$-\frac{d^2u}{dx^2} - u + x^2 = 0;$$

a) Using the weighted functions, $\phi_0 = 0$ and $\phi_i = x^i(1 - x^i); i = 1,2$ obtain the two parameter Rayleigh-Ritz solution subject to the boundary conditions,

$u(0) = 0, \mathrm{u}(1) = 0$ (10 marks)

b) Using the weighted functions, $\phi_0 = x$ and $\phi_1 = -x(2 - x)$ and $\phi_2 = x^2\left(1 - \frac{2}{3}x\right)$ obtain the two parameter Garlerkin solution subject to the boundary conditions

$u(0) = 0, u'(1) = 0$ (10 marks)