MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF EDUCATION (ARTS AND SCIENCE)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF ARTS

SMA 260 - PROBABILITY AND STATISTICS I

DATE: 19/01/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

1. *Answer* ***Question*** **1** *and any other* ***two*** *questions.*
2. *Each question must start on a new page;*
3. *You must have a Scientific Calculator and Statistical Tables for this paper:*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) (i) Outline **four** characteristics of the normal probability distribution. (2 marks)

(ii) If x is a discrete random variable, then by considering its variance and using the techniques of moments, show that

$$\sum [(x-\mu)^2] = \sum x^2 f(x) - \mu^2$$ (3 marks)

b) A discrete random variable *x* has a probability distribution function given by:

x	0	1	2	3	4	5	6
$f(x)$	0.06	0.15	0.30	0.22	0.14	0.08	0.05

(i) Show that the function $f(x)$ is a probability mass function (p.m.f.). (2 marks)

(ii) Determine the mean and the variance of the random variable x in the probability distribution. (5 marks)

c) A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{1}{40}(2x+3) \ ; & \text{for} \quad 0 \leq x \leq 5. \\ 0 \ ; & \text{elsewhere} \end{cases}$$

(i) Show that the function $f(x)$ is a probability density function (p.d.f.).

(ii) Determine the median of the random variable x (5 marks)

d) An international organisation claims that 60% of the secondary school leavers in Kenya are not able to communicate orally in English. A random sample of 12 secondary school leavers was selected in Kenya. Assuming that this claim is true, determine the probability that among these secondary school leavers, the following are not able to communicate orally in English:

(i) between 4 and 6 inclusive;

(ii) at least 3 school leavers. (7 marks)

e) A discrete random variable x has a Poisson probability distribution with probability mass function (p.m.f.) given by:

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & \text{for} \quad x = 0, 1, 2, 3, \ldots\ldots, n. \\ 0 \ ; & \text{elsewhere} \end{cases}$$

Prove that the mean of x and the variance of x are both equal to λ (6 marks)

QUESTION TWO (20 MARKS)

A continuous random variable x has a probability function given by:

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) \ ;\text{for} \ 1 \leq x \leq 5. \\ 0 \quad \text{elsewhere} \end{cases}$$

a) Show that the function $f(x)$ is a probability density function (p.d.f.). (3 marks)

b) Determine the following measures about the random variable x in the probability distribution:

(i) the mode of x ;

(ii) the mean of x ;

(iii) the variance of x . (13 marks)

c) Determine the following probability from the distribution:

$P(2 \leq x \leq 4)$. (4 marks)

QUESTION THREE (20 MARKS)

a) Outline **three** characteristics of the binomial probability distribution. (3 marks)

b) It is known that 2% of all the items coming out of a production process are defective and do not meet the industry specifications. Based on this proportion, determine the probability that among 450 items randomly selected from the production process, the following number will be defective:

i. between 5 and 8 items inclusive; (4 marks)

ii. at least 3 items. (4 marks)

iii. Show that in conformity with the property of the Poisson probability distribution, the mean and the variance are equal or approximately equal in this distribution. (3 marks)

c) A continuous random variable x has a uniform probability distribution with probability density function (p.d.f.) given by:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for} \quad a \leq x \leq b \qquad -\infty < a < b < \infty \\ 0 \; ; & \text{elsewhere} \end{cases}$$

Derive the mean of x and the variance of x. (6 marks)

QUESTION FOUR (20 MARKS)

The marks scored in Mathematics by students who sat for KCSE examination in certain year has been found to be normally distributed with mean 52 marks and standard deviation 13 marks.

(a) (i) Suppose the pass-mark is set at 42 marks, determine the proportion of the students who will pass. (2 marks)

(ii) Determine the proportion of the students who will score a Credit grade if a Credit is assigned for marks between 60 and 74. (5 marks)

(b) (i) If the top 75% of the students are supposed to pass this examination, determine the mark which should be set as the pass-mark to achieve this. (3 marks)

(ii) Suppose academic grades are awarded based on the following criteria:

- *Fail* to the bottom 20%,
- *Pass* to the next 35%,
- *Credit* to the next 30%,
- *Distinction* to the top 15%.

Determine the lower and upper limits of the range of the marks for each grade. (10 marks)

QUESTION FIVE (20 MARKS)

a) A continuous random variable x has an exponential probability distribution with probability density function (p.d.f.) given by:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} \;; & \text{for} \quad 0 \leq x \leq \infty, \qquad \lambda > 0. \\ 0 \quad \text{elsewhere} & \end{cases}$$

Show that the variance of x is given by: $var(x) = \frac{1}{\lambda^2}$ (10 marks)

b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x \;; & \text{for} \quad 0 \leq x \leq 1. \\ 0 \quad \text{elsewhere} & \end{cases}$$

(i) Determine the probability density function of a continuous random variable $y = 8x^3$ using the change of variable technique. (5 marks)

(ii) Hence determine the following:

I. the mean of y ;

II. the probability $P(Y > 4)$; (5 marks)