MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 400: TOPOLOGY

DATE: **TIME:**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 MARKS (COMPULSORY)

a) Let $X = \{1,2,3,4\}$ and $\rho = \{\emptyset,\ X,\ \{1\},\ \{2\},\ \{1,2\},\ \{1,2,3\},\ \{2,3\}\ \}$. Let $E = \{1,2,3\}$. Find the set of all limit points of E. (4 marks)

b) Give the definition of a topological space. (4 marks)

c) Let (X, ρ) be a topological space show that if $A, B \subset X$ and $A \subset B$ then $A^d \subset B^d$ (4 marks)

d) Let X be a non void set ρ_d be a discrete topology in X. If $E \subset X$ find E^d (the set of all limit points of E). (5 marks)

e) Let $X = \{i, j, k, l\}$. Let $S = \{\{i\}, \{j, k\}, \{k, l\}\}$ be a collection of subsets of X. Give the topology generated by the set S. (5 marks)

f) Let $X = \{i, j,\ k,\ l,\ m\}$ and $\rho = \{\emptyset,\ X,\ \{i\},\ \{j\},\ \{i, j\},\ \{k\},\ \{i, j, k\}, \{i, k\},\ \{j, k\}\ \}$.

g) Let $E = \{a, b, c\}$. Find:

i) The closure of E

ii) The interior of E (4 marks)

QUESTION TWO (20 MARKS)

a) Consider R with the usual topology and let $Y = (0,1) \cup \{2\}$ if $E = (\frac{1}{2}, 1)$ find the closure of E in the subspace Y. (6 marks)

b) Find the topology generated in R by the class of all closed sub-intervals of the type $(b, b + 1)$. (5 marks)

c) Find the topology generated by the sub-basis $\beta = \big\{\{a\},\ \{a,\ c,\ d\},\ \{b,\ c\}, \{c\}\big\}$, where $X = \{a, b, c, d\}$ (5 marks)

d) Let X be a non-empty set. Define the following topologies

i. Trivial topology

ii. Discrete topology (2 marks)

e) Define the neighborhood of a point $p \in X$ where X is a topological space. (2 marks)

QUESTION THREE (20 MARKS)

a) Given any collection of subsets of a nonvoid set X. Will this collection serve as a base for the topology? (5 marks)

b) If N_1 and N_2 are two neighborhoods of x, show that $N_1 \cap N_2$ is also a neighborhood of x. . (5 marks)

c) Consider the topology $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{1,3,4,5\}\}$ on $X = \{1,2,3,4,5\}$ and the subset

d) $A = \{b, d, e\}$ of X. Compute showing working out:

i. $Int(A)$ (3 marks)

ii. $Ext(A)$ (3 marks)

iii. $B'dary(A)$ (4 marks)

QUESTION FOUR (20 MARKS)

Let (X, ρ) be a topological space show that; If $y \in F^d$ then $y \in (F - \{y\})^d$. (5 marks)

a) Let (X, τ) be a topological space. Define a base and sub-base for the topology τ on X. (4 marks)

b) Let $X = \{x, y, z\}$ and $\tau = \{\{x, y\}, \{y, z\}, \emptyset, \{y\}, \{x, y, z\}\}$. Give the base and sub-base for the topology τ on X. (4 marks)

c) Given a subset A of a topological space, define what it means for p to be a limit point of A. (2 marks)

d) Consider the discrete topology τ on $X = \{1,2,3,4\}$. Find the sub-base and the bases for τ. (4 marks)

QUESTION FIVE (20 MARKS)

a) Let (X, ∂) be a metric space and A $\subset X$. Then prove that if P is a limit point of A every neighborhood of P contains infinitely many points distinct from P. (4 marks)

b) Let $X = \{a, b, c, d, e\}$ and $J = \{\emptyset, x\{a\}\{b, c\}\{b, c, d\}\{a, b, c\}\{a, d\}\{d\}\{a, b, c, d\}\}$ let $Y = \{a, b, c, d\}$,

i) find the relative topology on Y

ii) Define relative topology

iii) Find the closed set in relative topology (6 marks)

c) Define hereditary as used in topological spaces (2 marks)

d) T_0 and T_1 spaces are hereditary prove (4 marks)

e) Show that every metric space is a T_2 space. (4 marks)