MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 401: TOPOLOGY II

DATE: 21/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) COMPULSORY

a) Define the following terms

i) Finite intersection property

ii) Sequentially compact

iii) Compact spaces

iv) Separated sets

v) Locally compact sets (5 marks)

b) Prove that the open interval $A = (0, \frac{1}{2})$ on the real line $\mathbb{R}$ with the usual topology is not compact. (5 marks)

c) Consider the following topology on $X = \{a, b, c, d, e\}$ $\rho = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}, \{c\}\}$. Now $A = \{a, d, e\}$. Show that A is disconnected. (5 marks)

d) Prove that if X is sequentially compact then it is countably compact. (5 marks)

e) Prove that a set is disconnected if and only if it is not a union of two non-empty separated sets. (5 marks)

f) Prove that if X is compact then it is countably compact. (5 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Prove that if A and B are disjoint compact subsets of Hausdorff spaces X. Then ∃ open set G and H such that $A \subset G$ and $B \subset H$ and $G \cap H = \emptyset$ (6 marks)

b) Let $\mathbb{Z}$ be the set of integers. Is it sequentially compact. (3 marks)

c) Let $G \cap H$ be a disconnection of A. show that $G \cap A$ and $H \cap A$ are separated. (5 marks)

d) Let $G \cup H$ be a disconnection of A and B be a connected subset of A. Show that $B \cap G = \emptyset$ or $B \cap H = \emptyset$. (6 marks)

QUESTION THREE (20 MARKS)

a) Show that an infinite subset A of a discrete space X is not compact (7 marks)

b) Prove that a closed subset F of a compact set X is also compact. (7 marks)

c) Prove that a topological space X is compact if and only if $\{F_i\}$ of closed subsets of X satisfies the finite intersection property. (6 marks)

QUESTION FOUR (20 MARKS)

a) Define hereditary as used in topological spaces (2 marks)

b) $T_{0\ and}\ T_1$ spaces are hereditary prove (6 marks)

c) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (6 marks)

d) Prove that if $X, \rho)$is a topological space which is a T_2. Then every convergent sequence of points of X has a unique limit. (6 marks)

QUESTION FIVE (20 MARKS)

a) Prove that every compact subset of a Housdorff space is closed. (5 marks)

b) Show that if A and B are non-empty separated sets. Then, $A \cup B$ is disconnected (5 marks)

c) Show that every metric space is hausdorff space (5 marks)

d) Prove that if F is closed subset of a compact space X, then F is also compact. (5 marks)