# MACHAKOS UNIVERSITY

**University Examinations 2019/2020 Academic Year**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF MATHEMATICS AND STATISTICS**

**………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR**

**BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING**

**BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**SST 103: LINEAR ALGEBRA**

**DATE:** **TIME:**

**INSTRUCTIONS**

**Answer ALL the questions in Section A and ANY TWO Questions in Section B**

## SECTION A

### QUESTION ONE (30 MARKS) (COMPULSORY)

a) Find the cross product of the following vectors $(2,4,-1)$ and $(3,6,1)$ (3 marks)

b) Reduce the following matrixes to be reduced echelon form

$$A=\begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$ (3 marks)

c) Solve the following simultaneous equation using inverse matrix method

$$2x-y=4$$

$$3x+2y=6$$ (3 marks)

d) If $\begin{vmatrix} y & 5 \\ y & 2y+1 \end{vmatrix}=4$ find y (3 marks)

e) Calculate the cross product of the vectors $\vec{u}=(1, 2, 3)$ and $\vec{v}=(-1, 1, 2)$. (3 marks)

f) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$ (4 marks)

g) Show that the vector (11,3,-8) is a linear combination of the (1,1,0) and (2,1,-1)

h) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, −1). (4 marks)

i) Determine if (1,2,3,1), (2,2,1,3) and (-1,2,7,3) is linearly dependent (4 marks)

j) Find the angle between $u = 2i$ +2j+2k and $v = i + j + k$ (3 marks)

## SECTION B

### QUESTION TWO (20 MARKS)

**a)** Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$ (4 marks)

b) Transpose the following matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 4 & 7 \end{pmatrix}$$ (2 marks)

c) Find the minors, cofactors and adjoint of the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$ (4 marks)

d) Find the inverse of the following matrices

$$\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ (5 marks)

e) Reduce the following matrix into echelon form

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{pmatrix}$$ (5 marks)

**QUESTION THREE (20 MARKS)**

a) Solve by crammer's Rule

$$x - 2y + 3z = 10$$
$$3x - 6y + z = 22$$
$$-2x + 5y - 2z = -18$$

(5 marks)

b) Determine the value of 'a' so that the following systems in unknown $x$, $y$ and $z$ has

i. No solution

ii. More than one solution

iii. Unique solution

$$x - 3z = -3$$
$$2x + ay - z = -2$$
$$x + 2y + az = 1$$

(6 marks)

c) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$
$$x_1 + 2x_2 + 3x_3 = 10$$
$$2x_1 + x_2 + x_3 = 6$$

(4 marks)

d) Solve the simultaneous equation using the gauss-Elimination method

$$2x_1 - 4x_2 + 6x_3 = 20$$
$$6x_1 - 12x_2 + 2x_3 = 44$$
$$-4x_1 + 10x_2 - 4x_3 = -36$$

(5 marks)

**QUESTION FOUR 20 MARKS**

a) Define a vector space (6 marks)

b) Show that W= $\{(x, y) \ / \ x = 2y\}$ is a subspace for $R^2$. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)

**QUESTION FIVE 20 MARKS**

a) Show that *u*, v, and *w* are linearly independent

$u = (6, 2, 3, 4)$

$v = (0, 5, -3, 1)$

$w = (0, 0, 7, -2)$ (5 marks)

b) Write $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrices

$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}$ (5 marks)

c) Calculate the value of k for the vectors $\vec{u} = (1, k)$ and $\vec{v} = (-4, k)$ knowing that they are orthogonal. (5 marks)

d) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent. (5 marks)