MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ARTS (PUBLIC ADMINISTRATION)

APP 102: QUANTITATIVE TECHNIQUES FOR POLICY MAKERS

DATE: 27/1/2022 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question one And Any Other Two Questions.

QUESTION (COMPULSORY) (30 MARKS)

a) Solve the following equation: $\frac{3}{2}x + \frac{1}{3} = \frac{1}{4}x - \frac{1}{6}$ (2 marks)

b) Find the derivative of $f(x) = x^3 + 2x - 1$ using first principles or the three-step rule i.e. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, where $h \neq 0$ (4 marks)

c) If $cot\alpha = 1$, find $tan \propto, sin \propto, sec \propto, cosec \propto$ (4 marks)

d) Solve each system of equations by using A^{-1}:

$-2x - 3y = 1$

$3x + 4y = -1$ (4 marks)

e) Find $\lim_{x\to -1} \frac{x^2-1}{x^2-2x+1}$ (4 marks)

f) Evaluate $\int_1^3 (t^2 - 2t + 3)dt$ (4 marks)

g) Define a Markov chain, a one-step transition probability with an example having a state space (3 marks)

h) Maximize the objective function $P = x + 2y$ subject to the constraints: $y \leq 3, x + y \leq 5, x - 2y \leq 2, x \geq 0, y \geq 0$. Draw and find a feasible point or a feasible solution (5 marks)

QUESTION TWO (20 MARKS)

a) Find the first three terms and the 10^{th} term of each infinite sequence whose nth term is given by $a_n = \frac{(-1)^n}{(n+1)(n+2)}$ (4 marks)

b) Find the sum of each arithmetic series $\sum_{n=3}^{n=15}(-0.1n + 1)$ (5 marks)

c) Find the sum of each infinite series $-9.9 + 3.3 - 1.1 + \cdots$ (5 marks)

d) If Kshs 1000 is deposited at the end of each month for 30 years in a retirement account earning 9% compounded monthly, then what is the value of this annuity immediately after the last payment? (6 marks)

QUESTION THREE (20 MARKS)

a) Find the derivative of the function $f(x) = (x^2 - 1)\frac{x^3+3x^2}{x^2+2}$ (5 marks)

b) Use the second derivative test to determine the nature of the turning points of the function $f(x) = x^4 + 4x^3 - 1$ (5 marks)

c) Evaluate the integral $\int_0^2 x\sqrt{x^2 + 1}\,dx$ (5 marks)

d) The velocities of 2 runners are given by $f(t) = 10kmh\ and\ g(t) = (10 - sint)kmh$. Find and interpret the integrals $\int_0^{\pi}[f(t) - g(t)]dt$ and $\int_0^{\pi}[g(t) - f(t)]dt$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following system using Cramer's rule:

$x + y + z = 6$

$x - y + z = 2$

$2x + y + z = 7$ (8 marks)

b) Find AB and BA if $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ (4 marks)

c) Let $f(x) = x - 2, g(x) = x^2 + 1, h(x) = \frac{x+1}{3}$. Find $f\left[g\left(h(2)\right)\right]$ (4 marks)

d) Determine all functions for which the given function $f(x, y) = \sqrt{9 - x^2 - y^2}$ is continuous (4 marks)

QUESTION FIVE (20 MARKS)

(a) Consider the University undergraduate system in the 8-4-4 system. Suppose individuals have a 15% chance of repeating each of the years. From experience, 75% of the 1^{st}, 2^{nd} and 3^{rd} years respectively move to the next higher grade and that 80% of IV years pass the examination.

i. Represent this as a transition matrix in Canonical form (3 marks)

ii. Find the probability of leaving the system in 2 years (4 marks)

iii. What is the probability that an individual in grade I will be in the same grade after 3 years? (3 marks)

(b) Use simplex method to maximize $P = 4x + 3y$ subject to $-x + y \leq 4; x + 2y \leq 14; 2x + y \leq 16; x, y \geq 0$ (10 marks)