# MACHAKOS UNIVERSITY 

## University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS
FIRST YEAR FIRST SEMESTER EXAMINATION FOR
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
SAC 100: PRINCIPLES OF ACTUARIAL SCIENCE
DATE: 1/2/2022
TIME: 8:30-10:30 AM

INSTRUCTION: ANSWER QUESTION ONE [COMPUSLSORY] AND TWO OTHER QUESTIONS

QUESTION ONE (COMPULSORY) (30 MARKS)
a) Define the following terms.
i. Term assurance. (2 marks)
ii. Static risk.
b) Mention three types of systematic risk.
c) Calculate the following probability on the basis of ELT15[Males]

The probability that a person aged 60 dies within the first five years after retiring at age 65.
d) Show that

$$
\ddot{a}_{\bar{n} \mid}=(1+i) a_{\bar{n} \mid}
$$

e) Find the annual effective interest rate equivalent to a nominal interest rate of $12 \% p . a$ convertible four monthly .
f) Calculate the total present value as at $1^{\text {st }}$ June 2008 of payments of $£ 100$ on 1st January 2009 and $£ 200$ on $1^{\text {st }}$ May 2009, assuming a rate of interest of $12 \%$ p. a convertible quarterly.
g) Calculate the accumulated value of $£ 6.34$, assuming a force of interest of $9 \%$ p.a, after 3 months.

## QUESTION TWO (20 MARKS)

a) Calculate the effective annual rate of interest for a transaction in which $£ 100$ is invested for 24 months and another $£ 100$ for 12 months (starting 12 months after the first investment) to give a total of $£ 350$.
b) Project R delegates all the development work to outside companies. The estimated cash flows for project R are: (last column indicates expenditure)

| Beginning of year 1 | 150,000 dollars | Contractor fees |
| :--- | :--- | :--- |
| Beginning of year 2 | 250,000 dollars | Contractor fees |
| Beginning of year 3 | 250,000 dollars | Contractor fees |
| End of year 3 | $1,000,000$ dollars | Sales |

## Table 1

Project $S$ carries out all the development work in-house by purchasing the necessary equipment and using the company's own staff. The cash flows for the project S are: (last column indicates expenditure)

| Beginning of year 1 | 325,000 dollars | New equipment |
| :--- | :--- | :--- |
| Throughout year 1 | 75,000 dollars | Staff cost |
| Throughout year 2 | 90,000 dollars | Staff cost |
| Throughout year 3 | 120,000 dollars | Staff cost |
| End of year 3 | $1,000,000$ dollars | Sales |

Table 2
The staff cost can be estimated to be paid uniformly throughout the year.
i) Evaluate the NPV for the project R and S using a risk discount rate of $20 \% \mathrm{pa}$ ?
ii) Using the NPV as a criterion, which project is preferable?
ii) Using the NPV as a criterion, which project is preferable? . (2 marks)

## QUESTION THREE (20 MARKS)

a) You have hired as an actuary in an insurance company, state and explain five principles of insurance that you would implement.
b) $\quad$ Solve for n , if $\mathrm{P}=78.92, I=5, \mathrm{R}=125$ and $\mathrm{i}=10 \%$.

Where:

- $\quad \mathrm{P}$, is the price paid by the investor in return for a series of interest payment.
- I, interest payment payable at the end of each year of the next n years.
- n , number of years.
- R , redemption payment at the end of n years.


## QUESTION FOUR (20 MARKS)

a) Differentiate between fundamental and particular risks.
b) A company is interested in estimating policy lapse rates by age. It conducts an investigation into this, which lasts for the whole of the calendar year 2003. The investigation collects the following data for a sample of policies which are funded by annual premiums:

- the age last birthday of the policyholder when the policy was taken out;
- The number of premiums the policyholder paid before the policy lapsed. In addition, the number of policies in-force on 1 January each year is available, classified by age x last birthday and years t elapsed since 1 January 2003, $\left(P_{x, t}^{*}\right)$.
i. State the rate interval in this investigation.
(1 mark)
ii. Derive an expression for the exposed-to-risk in terms of $\left(P_{x, t}^{*}\right)$, stating any assumptions you make.
(13 marks)
iii. Comment on the reasonableness or otherwise of the assumptions you made in your answer to part (ii).


## QUESTION FIVE (20 MARKS)

a) Describe the three methods of graduation.
b) A property developer is constructing a block of offices. It is anticipated that the offices will take six months to build. The developer incurs costs of $£ 40$ million at the beginning of the project followed by $£ 3$ million at the end of each month for the following six months during the building period. It is expected that rental income from the offices will be $£ 1$ million per month, which will be received at the start of each month beginning with the seventh month. Maintenance and management costs paid by the developer are expected to be $£ 2$ million per annum payable monthly in arrear with the first payment at the end of the seventh month. The block of offices is expected to be sold 25 years after the start of the project for $£ 60$ million.
i) Calculate the discounted payback period using an effective rate of interest of $10 \%$ per annum.
ii) Without doing any further calculations, explain whether your answer to (i) would change if the effective rate of interest were less than $10 \%$ per annum.

