**MACHAKOS UNIVERSITY**

**SCHOOL OF PURE AND APPLIED STATISTICS.**

**DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE.**

**BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**SECOND YEAR: FIRST SEMESTER: SAC 203: ACTUARIAL MATHEMATICS I:**

**INSTRUCTION: ANSWER QUESTION ONE [COMPUSLSORY] AND TWO OTHER QUESTIONS**

**QUESTION ONE: 30 MARKS.**

a) Define the following terms.

i. Whole life assurance. **(2 marks)**

ii. Makeham's law. **(2 marks)**

b) Using ELT 15 (MALES) mortality, find the curtate expectation of life for, a 70 year old pensioner. **(3 marks)**

c) The force of interest $\delta(t)$ is:

$$\delta(t) = 0.005t + 0.0001t^2 \text{ For all t.}$$

**i.** At $t = 8$,calculate the accumulated value of an investment of £ 100 made at time$t = 0$. **(5 marks)**

ii. Calculate the constant annual effective rate of interest over the eight year period. **(3 marks)**

d) Using the PMA92C20 table for both lives calculate:

${}_{2}q^{1}_{65:65}$

**(5 marks)**

e) If $\mu_x$ takes the constant value 0.025, at all ages, calculate the age $x$ for which ${}_{x}p_0 = 0.5$ .what does this age represent? **( 5 marks)**

f) For a force of mortality $\mu_x$ that is known to follow Gompertz' law ,calculate the parameters B and C if $\mu_{50} = 0.017609$ and $\mu_{55} = 0.028359$ **(5 marks)**

## QUESTION TWO (20 MARKS)

a) A term assurance contract for a life aged 50 exact for a term of 10 years provides a benefit of £10,000 payable at the end of the year of death. Calculate the expected present value and variance of benefits payable under this contract.
. **(8 marks)**

Basis:

| | |
|---|---|
| Mortality: | AM92 Select |
| Interest: | 4% per annum |

b) An assurance contract provides a death benefit of £1,000 payable immediately on death, with a savings benefit of £500 payable on every fifth anniversary of the inception of the policy.
The following basis is used.

| | |
|---|---|
| Force of mortality: | $\mu_x = 0.05$ for all $x$ |
| Force of interest: | $\delta = 0.04$ |
| Expenses: | None |

Calculate the level premium payable annually in advance for life. **(12 marks)**

## QUESTION THREE (20 MARKS)

The force of interest $\delta(t)$ is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \leq 5 \\ 0.008t & 5 < t \leq 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

a) Calculate the present value of a unit sum of money due at time $t = 12$. .

. **(13 marks)**

b) Calculate the effective annual rate of interest over the 12 years. **(2 marks)**

**c)** Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$. .

. **(5 marks)**

**<u>QUESTION FOUR (20 MARKS)</u>**

$T_x$ denotes the future lifetime of a life aged $x$ .

**a)** Write down the probability density function of $T_x$ . **(1 mark)**

b) Using your answer to (a), show that **(10 marks)**

i. $\frac{\partial}{\partial s}\log\ {}_sp_x = -\mu_{x+s}$ and,

ii. ${}_tp_x = \exp\{-\int_0^t \mu_{x+s}\ ds\}$

c) In a certain population, the force of mortality is given by,

| | $\mu_x$ |
|---|---|
| $60 < x \leq 70$ | 0.01 |
| $70 < x \leq 80$ | 0.015 |
| $x > 80$ | 0.025 |

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83. **(9 marks)**

**<u>QUESTION FIVE (20 MARKS)</u>**

a) For the first 5 years after arrival in a certain country, lives are subject to a constant force of mortality of 0.005. Thereafter lives are subject to mortality according to ELT 15(Males) with an addition of 0.039221 to the force of mortality.

i. A life aged exactly 30 has just arrived in the country.

**I.** Show that the probability that the life will survive to age 35 is 0.97531. .

. **(2 marks)**

II. Find the probability that the life will survive to age 60. **(2 marks)**

**ii.** What is the probability that a life aged exactly 33 who has been in the country for 3 years will die between ages 50 and 51? (Assume that these lives will remain in the given country. **(4 marks)**

b) Suppose that Gompertz' law $\mu_x = Bc^x$ , holds for all $x \geq \alpha, c$ being greater than 1. Assume that $\mu_\alpha < \log c$ .

i. Give a formula for $s(x)$. **(4 marks)**

**ii.** Show that $l_x \mu_x$ attains a maximum when $\mu_x = \log c$ , and has no other stationary points. **(8 marks)**