MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF ECONOMICS AND FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMICS II

DATE: 18/8/2021 **TIME: 2:00 – 4:00 PM**

INSTRUCTIONS:

(i) Answer question one (COMPULSORY) and any other two questions

(ii) Do not write on the question paper

(iii) Show your workings clearly

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Compute the equilibrium price and equilibrium quantity for the following single market model. (2 marks)

$Q^d = 12 - P^2$

$Q^s = -6 + P^2$

b) The following input – output model shows the relationship between three sectors; agriculture, manufacturing and service sectors in the economy.

	Agriculture	**Manufacturing**	**Service**	**Final demand**	**Total**
Agriculture	10	30	10	50	100
Manufacturing	30	50	20	100	200
Service	10	20	20	50	100
Other factors	50	100	50	200	
Total factors	100	200	100		

Required:

i. Calculate the input – output coefficient and the Leontief matrices in this 3 sector industry (4 marks)

ii. Calculate the total output required for each industry when the final demand changes to 60, 120 and 60 for agriculture, manufacturing and service sectors respectively. (6 marks)

iii. State the assumptions of the input-output models. (2 marks)

c) Compute the derivative of the following functions

i. $y = 5e^{(2x^3 - 7x^2)}$ (3 marks)

ii. $y = 7^x$ (2 marks)

d) Find the consumer surplus, given the following demand function and equilibrium price.

$P = 50 - 0.5Q; \quad P_e = 30$ (4 marks)

e) Given the following constrained optimization problem

Maximize $Z = 2x + 6y - 3y^2$

s.t $\quad 2x - 2y^2 = 8$

Required: Find the critical values of x and y (3 marks)

f) Find the partial derivative of the following function (4 marks)

$$Z = \frac{4x^2 + 7}{x^2 + y^2}$$

QUESTION TWO (20 MARKS)

a) Compute the inverse of the following matrix (4 marks)

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{bmatrix}$$

b) Evaluate

$$\int_1^2 \frac{2 - 6x}{(2x - 3x^2)^3} dx$$

(5 marks)

c) If a firm faces the marginal cost schedule MC = $180 + 0.3q^2$ and the marginal revenue schedule MR = $540 - 0.6q^2$ and total fixed costs are 65, what is the maximum profit that the firm can make? (Assume that the second-order condition for a maximum is met.) (6 marks)

d) Given the following production function $Q = 40K^{0.5}L^{0.5}$

Prove Euler's Theorem (5 marks)

QUESTION THREE (20 MARKS)

a) Determine the profit maximizing level of output for a firm with the following total revenue and total costs functions. Check for second order condition.

$TR = 20lnQ$

TC = 5Q (4 marks)

b) TKK MATS a carpet manufacturing and exporting firm has to supply an order for 5000 pieces of wooden carpets of two varieties X and Y to KICC for conference rooms. The joint cost function for the two varieties of the carpets is given by $C = 100X^2 + 150Y^2$. The quantity of X and Y are not specified and so the firm is forced to supply any combination. The main goal of the firm is to minimize the cost of producing the carpets but meeting the demand by KICC.

Determine how many of each type of carpet the firm will produce to minimize cost. What will be the minimum cost? (6 marks)

c) Find the amount of capital formation over the period [2,6], given the following rates of net investment flows:

$I(t) = 12t^{\frac{1}{2}}$ (3 marks)

d) Determine whether matrix A below is singular or not. (3 marks)

$$\mathbf{A} = \begin{bmatrix} 8 & 1 & 3 \\ 4 & 0 & 1 \\ 6 & 0 & 3 \end{bmatrix}$$

e) Find f_{xy} and f_{yx} for each of the following functions (4 marks)

i. $f(x, y) = 3x^2 y + 6x^2 + 2y^3$

ii. $f(x, y) = e^{2y} y^3 + \ln(x^2 y^2)$

QUESTION FOUR (20 MARKS)

a) Given the demand for beans as $Q_b = 4850 - 5P_b + 1.5P_p + 0.1Y$

Income Y = 10,000

Price of beans P_b= 200

Price of peas P_p = 100

i. Compute own price elasticity of demand (4 marks)

ii. Find the income elasticity of demand (3 marks)

iii. Find the cross elasticity of demand (2 marks)

iv. How are the two commodities related and why? (2 marks)

b) The demand and supply functions for two interdependent goods are given by

$QD_1 = 400 - 5P_1 - 3P_2$

$QD_2 = 300 - 2P_1 - 3P_2$

$QS_1 = -60 + 3P_1$

$QS_2 = -100 + 2P_2$

Compute the equilibrium prices and quantities of good 1 and good 2 using Cramer's Rule. (6 marks)

c) A consumer is faced with the following constrained utility maximization problem

Maximize $U = 3xy$

Subject to $4x + 2y = 24$

Required:

Compute the critical values $\bar{x}$ and $\bar{y}$ (3 marks)

QUESTION FIVE (20 MARKS)

a) If a firm spends 650 dollars on fixed costs and its marginal cost function is specified as MC $= 82 - 16q + 1.8q^2$

Compute the firm's total cost function. (4 marks)

b) Three farmers A, B, and C bought the following units of wheel-barrows and ox-ploughs for the ploughing and planting season.

Farmers	**Wheel-Barrows**	**Ox-Ploughs**
A	50	7
B	30	4
C	40	5

The prices of a wheel-barrow and an ox-plough are KShs 3,000 and KShs 8,000, respectively.

i. Present the information in a matrix form (1 mark)

ii. Using matrix multiplication, find how much each one of the farmers spent on the wheel-barrows and ox-ploughs that they bought (3 marks)

c) You are given the following national income model:

$$Y = C + I + G$$

$$C = 120 + 0.8Y$$

$$I = 100 + 0.1Y$$

$$G = 300$$

Required:

i. Present the model in matrix format (2 marks)

ii. Using Cramer's rule, find the equilibrium values of Y, C and I. (6 marks)

d) Find the degree of homogeneity of the following production function and state the nature of returns to scale exhibited by the function.

$Q = AK^{2/5}L^{3/5}$ (4 marks)