



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF BUSINESS AND ECONOMICS

DEPARTMENT OF ECONOMICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF ECONOMICS & STATISTICS

BACHELOR OF ECONOMICS & FINANCE

BACHELOR OF ECONOMICS

BACHELOR OF ARTS

EES 300: MATHEMATICS FOR ECONOMISTS III

DATE: 9/8/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS:

Answer Question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 Marks)

- a) The marginal costs of two firms are given by the following functions:
1. $C'(Q) = 300e^{0.1Q+2}$ $TC = 3000$ when $Q = 0$
 2. $C'(Q) = 41 + 30Q - 5Q^2$ $TC = 3000$ when $Q = 0$
- i. Find the fixed cost for each firm. (4 marks)
 - ii. Find the total cost function $c(Q)$ for each firm (2 marks)
 - iii. What is the average costs of each firm (4 marks)
- b) Find the time path of capital $K(t)$ given the following rates of net investment flow functions
- i. $I(t) = 10t^{1/2} + 5$ $K(0) = 50$ (3 marks)
 - ii. $I(t) = 18t^{3/5} - 2$ $K(0) = 24$ (3 marks)
 - iii. For each of (i) to (ii) above, find the amount of capital formation over the interval $[2,5]$ (6 marks)
- c) Solve for x $\ln(\sqrt[3]{x+30}) = 2$ (4 marks)
- d) Verify that the following differential equation is exact and solve the equations
 $(yt^2 + y)dy + ty^2dt = 0$ (4 marks)

QUESTION TWO (20 MARKS)

- a) Derive general solution of the First order difference equations (FODE) (6 marks)
- b) Suppose you are given the following demand and supply functions
- $$Qd = \alpha - \beta P \quad (\alpha, \beta > 0)$$
- $$Qs = -\gamma + \delta P \quad (\gamma, \delta > 0)$$
- i. Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path $P(t)$ (6 marks)
- ii. What is the inter-temporal equilibrium price (2 marks)
- iii. What is the market clearing equilibrium price (2 marks)
- iv. Does the market have a dynamically stable equilibrium price? Explain (4 marks)

QUESTION THREE (20 MARKS)

- a) Find the general and definite solution to the following differential equations
- i. $\frac{dy}{dt} + 5y = 1 \quad y(0) = 7$ (3 marks)
- ii. $\frac{dy}{dt} - 14y = 10 \quad y(0) = 2$ (3 marks)
- b) Find the integral of the following
- i. $\int x^2(x^3 + 2)^{50} dx$ (3 marks)
- ii. $\int x(x + 2)^{20} dx$ (3 marks)
- c) Solve the following equations using matrix algebra (8 marks)
- $$2x + y + 3w = 15$$
- $$x + 3y + w = 10$$
- $$3x + 2y + 2w = 20$$

QUESTION FOUR (20 MARKS)

- a) Solve the following difference equations
- i. $y_{t+1} + 3y_t = 11 \quad (y_0 = 2)$ (3 marks)
- ii. $y_t = 3y_{t-1} + 4 \quad (y_0 = 1)$ (3 marks)
- b) For the general first order linear differential equation given as follows:
- $$\frac{dy}{dt} + ay = b$$
- i. Find the general solution (8 marks)

- ii. Decompose the general solution into two components, the complementary function y_c and the particular integral y_p and interpret each term. (4 marks)
- iii. Which of the two components in (ii) above determines whether the equilibrium is dynamically stable or not (2 marks)

QUESTION FIVE (20 MARKS)

- a) Find Y and r by Cramer's rule and inverse matrix, given the following IS-LM models (10 marks)

Goods Market

$$Y = C + I$$

$$Y = 200 + 0.2Y$$

$$I = 8 - 0.3r$$

Money Market

$$Md = 100 + 0.5Y + 0.25r$$

$$Ms = 120$$

$$Md = Ms$$

- b) Find the producer surplus when the inverse supply function is $p = 3Q + 5$ and the price $p = 65$ (6 marks)

- c) The growing value of GNP is given by:

$$GNP_t = GNP_0 e^{rt}$$

$$r = 1.5\%$$

- i. If $GNP_0 = 500$, find the value of GNP 10 years from now (2 marks)
- ii. If $GNP_0 = 1000$, after how many years will the GNP double? (2 marks)