



# MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 100 PRINCIPLES OF ACTUARIAL SCIENCE

DATE: 15/3/2022

TIME: 8.30-10.30 AM

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## INSTRUCTIONS:

Answer Question One and Any Other Two Questions

### QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Suppose that £860 is deposited in a savings account that pays simple interest at the rate of 5.375% per annum. Assuming that there are no subsequent payments to or from the account, find the amount finally withdrawn if the account is closed after:
- 6 months (2 marks)
  - 10 months (2 marks)
  - 1 year (2 marks)
- b) The effective compounding rate of interest per annum on a certain building society account is currently 7%, but in 2 years' time it will be reduced to 6%. Find the accumulation in 5 years' time of an investment of £4,000 in this account. (4 marks)
- c) If the nominal rate of interest is 12% per annum on transactions of term 2 years, calculate the accumulation of £100 invested at this rate over 2 years. (3 marks)
- d) The force of interest per unit time,  $\delta(t) = 0.12$  for all  $t$ . Find the nominal rate of interest per annum on deposits of term
- 7 days, (2 marks)
  - 1 month, (2 marks)

- iii. 6 months. (2 marks)
- e) An investor takes a loan at a commercial rate of discount of 18% per annum that is settled with a payment of £1,000 after:
- i. 3 months (5 marks)
- ii. 9 months (5 marks)
- Calculate the annual rate of discount and the effective rate of interest implied in each case.
- f) Define the term insurance (1 mark)

**QUESTION TWO (20 MARKS)**

- a) Define the following terms:
- i. Risk. (1 mark)
- ii. Contingency (1 mark)
- iii. Annuity Certain. (1 mark)
- iv. Annuity Due (1 mark)
- v. Annuity Immediate (1 mark)
- b) List and explain the basic characteristics of insurance. (8 marks)
- c) A borrower agrees to repay a loan of £3,000 by 15 annual repayments of £500, the first repayment being due after 5 years. Find the annual yield for this transaction. (7 marks)

**QUESTION THREE (20 MARKS)**

- a) The two sets of grandparents for a new-born baby wish to invest enough money immediately to pay *KSHS. 100,000* per year for four years toward college costs starting at age 18. Grandparents *A* agree to fund the first two payments, while Grandparents *B* agree to fund the last two payments. If the effective rate of interest is 6% per annum. Find the difference between the contributions of Grandparents *A* and *B*.  
(6 marks)
- b) It is known that  $a(t)$  is of the form  $at^2 + b$ . If *KSHS. 100* invested at time 0 accumulates to *KSHS. 172* at time 3. Find the accumulated value at time 10 of *KSHS. 100* invested at time 5. (4 marks)
- c) Show that  

$$\frac{d^3}{(1-d)^2} = \frac{(i-d)^2}{(1-v)}$$
 (5 marks)
- d) Show that the ratio of the accumulated value of  $I$  invested at rate  $i$  for  $n$  periods, to the accumulated value of  $I$  invested for  $n$  periods at rate  $r$ . Find an expression for  $r$  as a

function of  $i$  and  $j$ .  
marks)

(5

#### QUESTION FOUR (20 MARKS)

- a) An investor is considering whether to invest in either or both of the following instruments:

*Investment A:*

For a purchase price of £10,000 the investor will receive £1,000 per annum payable quarterly in arrears for 15 years.

*Investment B:*

For a purchase price of £11,000, the investor will receive an income of £605 per annum, payable annually in arrears for 18 years, and a return of his outlay at the end of this period.

The investor may lend or borrow money at 4% per annum. Would you advise him to invest in either loan, and, if so, which would be the more profitable? (10 marks)

- b) A mining company is considering an opencast project. It is estimated that the opencast site will produce 10,000 tonnes of ore per annum continuously for 10 years, after which period there will be an outlay of £300,000 to restore the land. The purchase price of the mining rights will be £1,000,000 and mining operations will cost £200,000 per annum, payable continuously. The company has insufficient funds to finance this venture, but can borrow the initial outlay of £1,000,000 from a bank, which will charge interest at 12% per annum effective; this loan is not for a fixed term, but may be reduced by repayments at any time. When the mining company has funds to invest, it will receive interest calculated at 10% per annum effective on its deposits. On the assumption that the price of ore is such that this project will just break even, determine (to the nearest month)
- how long the mining company will take to repay its bank indebtedness and (5 marks)
  - hence calculate this minimum ore price. (5 marks)

#### QUESTION FIVE (20 MARKS)

- a) A woman has an inheritance in a trust fund for a family members left by her recently deceased father that will pay \$ 50,000 at the end of each year indefinitely into the future . She has just turned 60 and does not think that this perpetuity-immediate meets retirement needs. She wishes to exchange the value of her inheritance in the trust fund for one which will pay her a 5 – year deferred annuity-immediate providing her a retirement annuity with

annual payments at the end of each year for 20 *years* following the 5 – *year* deferral period. She would have no remaining interest in the trust fund after 20 *payments* are made. If the trustee agrees to her proposal, how much annual retirement income would she receive? The trust fund is earning an annual effective rate of interest equal to 5%. Answer to the nearest dollar.

(10 marks)

- b) A worker aged 40 wishes to accumulate a fund for retirement by depositing \$3,000 at the beginning of each year for 25 years. Starting at the age 65 the worker plans to make 15 annual withdrawals at the beginning of each year. Assuming that all payments are certain to be made, find the amount of each withdrawal starting at age 65 to the nearest dollar, if the effective rate of interest is 8% during the first 25 years but only 7% thereafter.

(6 marks)

- c) Explain the two important ingredients in the study of contingent models. (4 marks)