MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SAC 304 PENSION MATHEMATICS

DATE: 26/7/2023   **TIME: 8.30-10.30 AM**

INSTRUCTION:
ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

SECTION A: (COMPULSORY)

QUESTION ONE (30 MARKS)

a)   Explain why the discount function for age 65 is V^{10} rather than $V^{10(1/2)}$ (1 mark)

b)   Describe the main features of an Accrued Benefit Funding Method used in connection with a defined benefit pension scheme.   (6 marks)

c)   A medium sized final salary scheme of a company in a developed country provides a pension of 1/50th of pensionable salary at normal retirement for each year of pensionable service. Pensionable salary is calculated annually on the scheme's anniversary (or at date of entry) as the member's basic salary less the State flat rate Pension at the same date. Describe, with formulae, how to analyse the salary experience of the scheme over the 3 years   since   the   previous   valuation.
         (5 marks)

d)   Write down an expression, using commutation functions, for the EPV of a lump sum of £150,000 paid immediately on the event of ill-health retirement, for an active pension scheme member aged exactly 48. Assume that ill-health retirement is only permitted before the member reaches his 60th birthday. Define all the symbols that you use   (4 marks)

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e) A pension scheme provides a pension of n 60 ths of final average salary payable on normal age retirement. However, n is limited to a maximum of 40 years. Write down a commutation function formula for calculating the total service liability for a member currently aged x , with a whole number of years' past service who joined the scheme:

    i.   after age 25, and   (4 marks)

    ii.   before age 25.   (3 marks)

f) Contributions to a pension scheme by employees are made at a rate of 5% of salary when aged under 35, 6% between ages 35 and 45, and 7 1 2% when aged 45 or over. Calculate the present value of the future contributions payable by a member aged exactly 30 who in the past year has received a total salary of £12,718. (7 marks)

SECTION B: ANSWER ANY TWO QUESTIONS (40 MARKS)

QUESTION TWO (20 MARKS)

A company pension scheme provides the following benefits for all members:

1. a pension on retirement (on grounds of ill-health or of age) of one-eightieth of final pensionable salary for each year of service (including fractions),

2. a lump sum on retirement of 3 times the annual pension,

3. on death in service, a lump sum of £30,000,

4. on withdrawal from service, a return of the employee's contributions, accumulated at 3% per annum compound.

Final pensionable salary is defined as the average annual salary in the three years immediately before retirement. All members who reach age 65 retire immediately.

Employees contribute to the scheme at the rate of 3% of salary, payable continuously. Salaries are revised continuously. The employer's contribution rate is assessed for each member separately, and is such that the prospective reserve for each new entrant is zero. Expenses are ignored.

a) Derive a formula, in terms of suitable commutation functions, for valuing benefit (1) above in respect of a new entrant aged x with annual salary rate SAL. (You need not define the service table functions). (4 marks)

b) In respect of a new entrant aged x with annual salary rate SAL, give formulae for valuing benefits (2), (3) and (4) above, using suitable commutation functions. (You need not derive

the formulae.) (c) Hence find a formula for the employer's contribution rate for a new member aged x and a starting salary rate of £10,000 p.a. (6 marks)

c) Using the basis given in the Formulae and Tables (and the supplement), find the value of each of the benefits (1), (2), (3) and (4) for a new entrant aged 45 with salary rate £10,000 per annum. (5 marks)

d) Hence or otherwise find the employer's contribution rate for this new member. (5 marks)

QUESTION THREE (20 MARKS)

a) List the three main classes of the benefit valuation methods. Give a valuation example for each example. (6 marks)

b) Consider a pension benefit to a life now aged 40 where the benefit is Ksh. 100,000 per month on a retirement due to attaining a mandatory retirement of 65. Assume pension benefit is not available for early retirement nor for other modes of retirement. Express the values of the benefit in each of the modes noted in part (i), explaining all the symbols used. (8 marks)

c) A pension scheme provides a pension on age retirement of 1/60th of final pensionable salary for each year of service, with part years counting proportionately. Final pensionable salary is defined to be the average salary over the 3 years prior to retirement. Members contribute 6% of their salaries to the pension fund. One member aged exactly 50 has 18 years of past service and earned £45,000 in the last year. Using the Pension Scheme Tables from the Actuarial Formulae and Tables, calculate:

    i.   the expected present value of this member's past service benefit (2 marks)

    ii.   the expected present value of this member's future service benefit (2 marks)

    iii. the expected present value of this member's future contributions. (2 marks)

QUESTION FOUR (20 MARKS)

a) The trustees of a medium size defined benefit scheme are reviewing the scheme's investment principles and strategy.

    i.   Outline the factors the trustees should consider when determining the investment principles and strategy. (6 marks)

ii. Describe (stating any assumptions together with any formulae as necessary) the discounted cash flow valuation method that could be used by the actuary to determine the value of the scheme's direct holdings of:

    I.   equities

    II.   property

    III.   fixed interest bonds   (6 marks)

b) Describe the characteristics of the investment products that are available as an alternative to direct investment. (5 marks)

c) List the items for which you will need to make assumptions in order to value the scheme's liabilities (3 marks)