MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SST 103: LINEAR ALGEBRA

DATE: 17/3/2022 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 Marks (Compulsory)

a) Show that $(A-B)(A+B)=A^2-B^2$ iff $AB=BA$ i.e A & B commute (4 marks)

b) Given $A=\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}$, $B=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$, show that $(AB)^T=B^TA^T$. (3 marks)

c) Determine the values of x for which the determinant is zero where

$$\begin{pmatrix} x-2 & 4 & 3 \\ 1 & x+1 & -2 \\ 0 & 0 & x-4 \end{pmatrix}$$

(4 marks)

d) Reduce $\begin{bmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 1 \\ 4 & -3 & 11 & 2 \end{bmatrix}$ to reduced row-echelon (canonical) form. (3 marks)

e) Find the angle between $u=i+j+k$ and $v=i+j-k$ (3 marks)

f) Solve the following simultaneous equation using inverse matrix method

$$2x-y=4$$

$$3x+2y=6$$

(3 marks)

g) If $\begin{vmatrix} y & 5 \\ y & 2y+1 \end{vmatrix}$=4 find y (3 marks)

h) Show that the vector (11,3,-8) is a linear combination of the (1,1,0) and (2,1,-1) (4 marks)

i) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3,2,0) and (2,1, −1). (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

a) Let $a = 3i - j + k$ and $b = i + 2j - k$

i. Find $\vec{a}\times\vec{b}$

ii. Show that $\vec{a}\times\vec{b}$ is orthogonal to $\vec{a}$

iii. Show that $\vec{a}\times\vec{b}$ is orthogonal to $\vec{b}$ (6 marks)

b) Let $A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix}$

i. Find det A

ii. Find Adj (A)

iii. Hence find A^{-1} (6 marks)

c) Find the cross product of the following vectors $(2,4,-1)$ and $(3,6,1)$ (4 marks)

d) Reduce the following matrixes to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix}$$ (4 marks)

QUESTION TTHREE (20 MARKS)

a) Solve using Crammer's rule.

$$x_1 + 3x_2 + 2x_3 = 3$$
$$2x_1 + 4x_2 + 2x_3 = 8$$
$$x_1 + 2x_2 - x_3 = 10$$

(5 marks)

b) Solve the following equations using inverse matrix method

$$x+2y+z=4$$
$$3x-4y-2z=2$$
$$5x+3y+5z=-1$$

(5 marks)

c) Solve for x, y and z using Gauss-Jordan method

$$x+y+z=5$$
$$x+2y+3z=10$$
$$2x+y+z=6$$

(5 marks)

d) Solve by elimination method

$$x_1+x_2+x_3=5 \quad \cdots$$
$$x_1+2x_2+3x_3=10\cdots$$
$$2x_1+x_2+x_3=0\cdots$$

(5 marks)

QUESTION FOUR (20 MARKS)

a) Define a vector space (6 marks)

b) Show that W= $\{(x, y) \,/\, x = 2y\}$ is a subspace for R^2. (4 marks)

c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)

d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3,5,-6)$ (5 marks)

QUESTION FIVE (20 MARKS)

a) Show that $w = \{\ (x, y, z) | x + y + z = 0)\}$ is a subspace of $\mathbb{R}^3$ (5 marks)

b) Calculate the value of k for the vectors $\vec{u}$ = (1, k) and $\vec{v}$ = (–4, k) knowing that they are orthogonal. (5 marks)

c) Show that the vectors u=(1,-1,0) v=(1,3,-1) and w=(5,3,-2) are linearly dependent. . (5 marks)

d) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

(5 marks)