MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SMA 363 TESTS OF HYPOTHESIS I

DATE: 25/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

a) Explain briefly what is meant by hypothesis testing as used in statistics. (2 marks)

b) Outline the general procedure used for hypothesis testing. (4 marks)

c) Differentiate between Type I and Type II errors as used in hypothesis testing. (4 marks)

d) Use the Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_0 < \theta_1$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. (7 marks)

e) A new gene has been identified that makes carriers particularly susceptible to a particular degenerative disease. In a random sample of 250 adult males born in Kenya, 8 were found to be carriers of the disease. Test whether the proportion of adult males born in Kenya carrying the gene is less than 10%. (5 marks)

f) A physician claims that the variance in cholesterol levels of adult men in a certain laboratory is at least 100. A random sample of 25 adult males from this laboratory

produced a sample standard deviation of cholesterol levels as 12. Test the physicians claim at 5% level of significance. (4 marks)

g) Differentiate between a Most powerful test and the Uniformly most powerful test. (4 marks)

QUESTION TWO (20 MARKS)

a) In a one-year investigation of claim frequencies for a particular category of motorists,

there were 150 claims from the 500 policyholders aged under 25 and 650 claims from

the 4,500 remaining policyholders. Assuming that the number of claims made by individual motorists in each category has a Poisson distribution, test at the 1% level whether the claim frequency is the same for drivers under age 25 and over age 25. (5 marks)

b) Let $X_1, \dots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive the most powerful test for testing $H_0: \lambda = 3$ against $H_a: \lambda = 6$. (7 marks)

c) If $X \geq 1$ is the critical region for testing $H_0: \theta = 2$ versus $H_1: \theta = 1$ on the basis of a single observation from the population

$$f(x, \theta) = \theta \exp(-\theta x),\ 0 \leq x \leq \infty$$

Determine the type I and type II errors. (8 marks)

QUESTION THREE (20 MARKS)

a) The average blood pressure for a control group C of 10 patients was 77.0mmHg. The average blood pressure in a similar group T of 10 patients on a special diet was 75.0mmHg. Carry out a statistical test to assess whether the variance in the two populations can be considered to be equal. Take $\sum_{i=1}^{10} C_i^2 = 59{,}420$, $\sum_{i=1}^{10} T_i^2 = 56{,}390$ and $\alpha = 0.05$. (6 marks)

b) Suppose the time to failure X for a piece of equipment is an exponential random variable with parameter λ. Given a random sample $X_1, \dots, X_n$ of lifetimes derive the most powerful test to test $H_0: \lambda = 0.01$ versus $H_a: \lambda = 0.04$. (4 marks)

c) Let $X_1, \dots, X_n$ be a random sample from a $N(\mu, \sigma^2)$. Assume that σ^2 is known. Determine an appropriate likelihood ratio test to test $H_0: \mu = \mu_0$vs $H_a: \mu \neq \mu_0$at level α. (10 marks)

QUESTION FOUR (20 MARKS)

a) State two asymptotic properties of the likelihood ratio tests? (2 marks)

b) The intelligence quotients (IQs) of 17 students from one area of a city showed a sample mean of 106 with a sample standard deviation of 10 whereas the IQs of 14 students from another area chosen independently showed a sample mean of 109 with a sample standard deviation of 7. Is there a significant difference between the IQs of the two groups at $\alpha = 0.02$? Assume that the population variances are equal. (8 marks)

c) It is desired to investigate the level of premium charged by two companies for contents

policies for houses in a certain area. Random samples of 10 houses insured by Company A are compared with 10 similar houses insured by Company B. The premiums charged in each case are as follows:

Company A	117	154	166	189	190	202	233	263	289	331
Company B	142	160	166	188	221	241	276	279	284	302

For these data, $\sum A = 2{,}134$, $\sum A^2 = 494{,}126$ $\sum B = 2{,}259$ $\sum B^2 = 541{,}463$

Test whether the level of premiums charged by Company B was higher than that charged by Company A. Take $\alpha = 0.05$ (10 marks)

QUESTION FIVE (20 MARKS)

a) Explain briefly the procedure for applying Neyman-Pearson lemma. (2 marks)

b) Ten persons were appointed in an officer cadre in an office. Their performance was noted by giving a test and the marks were recorded out of 100. They were given 4 months training and a test was held and marks were recorded out of 100.

Before	80	76	92	60	70	56	74	56	70	56
After	84	70	96	80	70	52	84	72	72	50

Do the data provide sufficient evidence to support the claim that the employees benefited from the training? Take $\alpha = 0.05$. (8 marks)

c) A correlation coefficient of 0.72 is obtained from a sample of 29 pairs of observations. Can the sample be regarded as drawn from a bivariate normal population with correlation coefficient 0.8? Take $\alpha = 0.05$.
(10 marks)