MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 332: METHODS OF APPLIED MATHEMATICS I

DATE: 25/8/2022 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30MARKS) COMPULSORY

a) State the equation representing each of the given canonical forms of 2nd order PDEs: hyperbolic, parabolic and elliptic forms. (3 marks)

b) Decompose the Laplace equation $u_{xx} + u_{yy} = 0$, $0 < x, y < L$ into ordinary differential equations (ODEs) in variables X and Y. (5 marks)

c) Sate the Fourier Cosine series expansion of a function $f(x)$ defined over the interval, $-\pi < x < \pi$. (2 marks)

d) Show that the Fourier coefficient b_n of the Fourier series, you have defined in b) above is given by; $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx, \quad n = 1,2,3,\dots$ (5 marks)

e) Given that $u = u(x, y)$ and the change of variables $\theta = \theta(x, y), \eta = \eta(x, y)$ such that $u_y = u_\theta \theta_y + u_\eta \eta_y$, show that;

$u_{yy} = u_{\theta\theta}\theta_y^2 + 2u_{\theta\eta}\theta_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\theta\theta_{yy} + u_\eta\eta_{yy}$ (5 marks)

f) Use the Convolution theorem to determine the inverse Laplace transform of the quotient $\frac{6}{(s^2+1)(s^2+9)}$ (5 marks)

g) Show that the Laplace transform $\frac{d^2y}{dt^2}$ is given by $s^2\bar{y}(s) - sy(0) - y'(0)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Classify the 2nd order partial differential equation (PDE) $y^2u_{xx} + x^2u_{yy} = 0$ and transform it to canonical form. (13 marks)

b) Obtain the Fourier series expansion for the function

$$f(x) = x^2, \quad -\pi < x < \pi$$
$$f(x) = f(x+2\pi)$$

(7 marks)

QUESTION THREE (20 MARKS)

a) Use partial fractions to determine the inverse Laplace transform of the quotient:

$$\frac{6+s^3}{(s+2)s^3}$$

(8 marks)

b) Solve the following 2nd differential equation using Laplace transform method;

$y'' + 4y' + 5y = 8\sin t$ subject to $y(0) = 0,\ y'(0) = 0$ (12 marks)

QUESTION FOUR (20 MARKS)

a) A heavy-duty wire of length a that is fixed at $x = 0$ and free to move at the other end $x = a$ is initially at rest and subjected to a constant force. Assuming that the wire's displacement $u(x,t)$ happens only in the x direction and $u_x(a,t) = D$ where D is the displacement per unit length.

i. Show that the Laplace transform of the associated wave equation is given by the ODE $\frac{d^2U(x,s)}{dx^2} = \frac{s^2}{c^2}U(x,s)$ (5 marks)

ii. Show that the solution to the ODE given in 4 a) i) above is

$U(x,s) = \frac{cD\sinh(sx/c)}{s^2\cosh(sa/c)}$ (10 marks)

b) Determine the Laplace transform of $f(x) = \sin ax$. (5 marks)

QUESTION FIVE (20 MARKS)

a) A thin bar of length π units is place in boiling water (temperature 100^oC). After reaching 100^oC throughout, the bar is removed from the boiling water. With the lateral sides kept insulated, suddenly, at time $t = 0$, the ends are immersed in a medium with constant freezing temperature 0^oC. Taking c=1, find the temperature $u(x,t)$ for $t > 0$ using the separation of variables method

(10 marks)

b) Obtain the general solution to the 2nd order PDE $yu_{tt} - k^2yu_{yy} = 2k^2u_r$ where k is a constant. (10 marks)