MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF EDUCATION (SPECIAL NEEDS EDUCATION)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

SMA 432: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 22/8/2022 **TIME: 2.00-4.00 PM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30MARKS) COMPULSORY

a) Under what conditions can the Lagrange equation $Pp+Qq=R$ be classified as: Linear, Semi-Linear and Quasi-Linear. (4 marks)

b) Sketch a diagram to illustrate the notion of orthogonal trajectories on some surface $f(x,y,z)=0$ (2 marks)

c) Find a partial differential equation arising from $z=ax^2+by^2+ab$ (4 marks)

d) Find the complete solution to $(y-z)\frac{\partial z}{\partial x}+(x-y)\frac{\partial z}{\partial y}=z-x$ (5 marks)

e) Find the integral curves of the equations:

i. $$\frac{dx}{x+y}=\frac{dy}{x+y}=\frac{dz}{-(x+y+2z)}$$ (5 marks)

ii. $y^2zp-x^2zq=x^2y$ (5 marks)

iii. $\dfrac{dx}{xz-y}=\dfrac{dy}{yz-x}=\dfrac{dz}{1-z^2}$ (5 marks)

QUESTION TWO (20 MARKS)

a) Show Pfaffian differential equation given by $z\left(x+y^2\right)dx+z\left(z+x^2\right)-xy\left(x+y\right)dz=0$ is integrable. (7 marks)

b) Find the solution of the equation $\left(x^2-y^2-z^2\right)p+2xyq=2xz$ (9 marks)

c) Eliminate the arbitrary constant from $z=ax^2-by^3+4ab$ (4 marks)

QUESTION THREE (20 MARKS)

a) Given that $u\equiv u(x,y,z)=c_1$, $v\equiv v(x,y,z)=c_2$ are solutions of $\dfrac{dx}{P}=\dfrac{dy}{Q}=\dfrac{dz}{R}$. Show that $F(u,v)=0$ is a general solution of the Lagrange's equation. (10 marks)

b) Find the integral curves of the equation $\dfrac{dx}{y(x+y)+az}=\dfrac{dy}{x(x+y)-az}=\dfrac{dz}{z(x+y)}$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider a surface $F(x,y,z)=0$ and let this surface intersect with the family of surfaces $G(x,y,z)=k$. Using Cramer's rule, derive the equations:

$P'=RF_y-QF_z, Q'=PF_z-RF_x, R'=QF_x-PF_y$ (10 marks)

b) Find the surface which intersects the surfaces of the system $z\left(x+y\right)=c\left(3z+1\right)$ and which passes through the circle $x^2+y^2=1$, z = 1 (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the general solution to the P.D.E.

$xp+yq=3z$ (6 marks)

b) Eliminate the arbitrary function from $\phi\left(x+y+z,\ x^2+y^2+z^2\right)=0$ (4 marks)

c) Find the integral surface of the equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$ (10 marks)