MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

DATE: 25/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

- Answer Question ONE and any other TWO questions.
- Mobile phones and any written material are prohibited in the examination room.
- No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.
- All answer booklets should be handed in at the end of the exam whether used or not.
- Programmable calculators are prohibited

QUESTION ONE (COMPULSORY, 30 MARKS)

a) Define the limit of a complex function $f(z)$ and hence evaluate $\lim_{z\to 2} f(z) = z^2 + 2z$ (3 marks)

b) Evaluate the points of discontinuity of $f(z) = \frac{z^2+1}{z^4-16}$ (3 marks)

c) Evaluate $\oint_c \frac{z+4}{z^2+2z+5} dz \; c: |z+1| = 3$, using Cauchy's theorem. (5 marks)

d) A point $3+bi$ on a z-plane is mapped onto the point $(11,c)$ on the w-plane by the mapping function $f(z)=2z^2+1$, find the values of b and c . (5 marks)

e) Solve the equation $z^4 - 16 = 0$. (5 marks)

f) Find $|z|^2$ given that $z = \frac{2+i}{3-2i}$ (4 marks)

g) Show that $\cos z = \cos x \cosh y - i \sin x \sinh y$ hence deduce $\cos iy = \cosh y$ (5 marks)

QUESTION TWO (20 MARKS)

a) Find all the values of z such that $e^z = 1 + i\sqrt{3}$ (5 marks)

b) Determine the singular points of the following function and residues at each point

$f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\oint_c \frac{z^2}{(z-1)^2(z+2)} dz$ where $c: |z| = 3$. (7 marks)

c) Show that $\cos^{-1} z = -iln(z + \sqrt{z^2 - 1})$. Hence find all solutions to the equation

$\cos z = \sqrt{2}$ (8 marks)

QUESTION THREE (20 MARKS)

a) Expand $f(z) = \frac{3}{z^2(z-3)^2}$ in a Laurent series at $z = 3$. (5 marks)

b) State the condition for integrability of $f(z)$. (1 mark)

c) State and prove Cauchy's integral formula and give the general expression for the n^{th}derivative of $f(z)$. (7 marks)

d) Integrate z^2 along the straight line OM (direct) and along an indirect path consisting of two straight line segments OL and OM, where O is the origin, M is the point $z = 3 + i$ and $L(3,0)$. Show that integral of z^2 along the two paths are equal. Hint: Sketch the region. (7 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate the points of discontinuity of

$f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z = 5}{z - i}$ (4 marks)

b) Find the fifth root of the complex number $-4 + 4i$ (6 marks)

c) Evaluate the following integral $\int_C \frac{e^{iz}}{z^2(z-2)(z+5i)} dz$ $|z| = 3$ using residue theorem.

(10 marks)

QUESTION FIVE (20 MARKS)

a) Find the residue of $f(z) \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles (6 marks)

b) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)dz$ along the curve $x = t + 1$, $y = 2t^2 - 1$ (8 marks)

c) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace equation. Hence determine the analytic function $f(z) = u + iv$ where u is given above. (6 marks)