MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATIONS (ARTS)

SMA 430 NUMERICAL ANALYSIS II

DATE: 31/8/2022 **TIME: 8.30-10.30 AM**

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION 1(30MKS)

a) Use Gauss elimination method with pivoting to solve the system of equations. (5 marks)

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

b) Determine the Eigen values and the corresponding Eigen vectors of the following system. (5 marks)

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

c) Apply scaled partial pivoting in solving the system (5 marks)

$$3.0x + y + 2.0z = 0$$
$$-2.5x - 0.5y - 11.0z = 0$$
$$1.8x + 2.8y + 4.0z = 16.1$$

d) Consider the following matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

i. Show that the matrix satisfies its own characteristics equation (5 marks)

ii. Determine A^{-1} (5 marks)

e) Use Gauss elimination method with pivoting to solve the system of equations. (5 marks)

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

QUESTION TWO (20 MARKS)

a) Consider the system

$$5x_1 + 5x_2 - x_3 = 6$$
$$x_1 + 6x_2 - 3x_3 = 4$$
$$2x_1 + x_2 + 4x_3 = 7$$

Use Jacobi's iterative method and perform 3 iterations to solve the system. (5 marks)

b) Apply power method to find the dominant Eigen value and Eigen vector of the matrix

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$ (5 marks)

c) Consider the differential equation $\frac{dy}{dx} = 2e^x y$, $y(0) = 2$. Calculate $y(0.2)$ using Adams Method by calculating $y(0.1)$, $y(0.2)$ *and* $y(0.3)$ using the Euler's modified formula.

(10 marks)

QUESTION THREE (20 MARKS)

a) Solve the system of equations using the Gauss- Elimination method with partial pivoting

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$ (5 marks)

b) Solve the following system below using the L-U decomposition method

$$\begin{aligned} -x_1 - 2x_2 - 4x_3 + x_4 &= -10 \\ 2x_1 + 7x_2 + 14x_3 + 4x_4 &= 26 \\ x_1 + 4x_2 + 9x_3 + 6x_4 &= 13 \\ 4x_1 + 10x_2 + 17x_3 - 5x_4 &= 43 \end{aligned}$$ (15 marks)

QUESTION FOUR (20 MARKS)

a) Use Runge-Kutta method of order four to solve the initial value problem (IVP)

$y' = y + \frac{1}{10}xy^2 \quad ; \; y(0) = 2$ at $x = 0.2(0.2)1$ (10 marks)

b) Find the second degree curve to the following data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(10 marks)

QUESTION FIVE (20 MARKS)

a) Let $f(x) = \ln x$ and $x_0 = 1.8$ for $h > 0$,evaluate $f'(x)$ (5 marks)

b) Calculate $\int_0^{\frac{1}{2}} \frac{x}{\sin x}\,dx$ using Romberg integration with step size $h = \frac{1}{16}$ (5 marks)

c) Consider the initial value problem (I .v. p) given by

$y' = 2x + 3y \quad ; \; y(0) = 1$

Use Taylors series second order method to get $y(0.4)$ with step size length $h = 0.1$

(10 marks)